

AN EXPERIMENTAL DERIVATION OF AN EXPANSIBILITY FACTOR FOR THE V-CONE and WAFER-CONE METERS

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1 INTRODUCTION

The V-Cone meter is a differential pressure device that has been developed and tested since 1985 and is now well understood and accepted as a viable flow measurement device. It is widely used in many industrial applications to measure a variety of fluids over a wide range of temperatures and pressures. The choice of the V-Cone can be based on the relatively short upstream lengths required and the ease of operation over a wide turndown ratio relative to an orifice plate.

Over the past 10 years the V-Cone has been increasingly accepted in the Gas Industry as a reliable metering device. Due to the fact that gas is a compressible fluid it is necessary to apply an expansibility correction factor, ε (or the Y factor in the USA), resulting in the well known mass flow equation:

$$q_m = \frac{C_d}{\sqrt{1-\beta^4}} \varepsilon \frac{\pi}{4} d^2 \sqrt{2\Delta p \rho} \quad (1)$$

Kinghorn [1] in his 1986 paper on this subject stated, "The expansibility coefficient, ε , compensates for the fact that changes in the pressure of the gas as it flows through the meter result in changes in its density. For nozzles and Venturi meters this can be computed on the assumption that the flow is adiabatic since it is constrained by the walls of the meter. For orifice plates, however, the three-dimensional expansion which takes place requires an empirical determination of values of the expansibility coefficient".

The expansibility factor equations for Venturi meters/nozzles and orifice plates are given by ISO 5167-1:1991 as:

$$\varepsilon = \left[\left(\frac{\kappa \tau^{2/\kappa}}{\kappa - 1} \right) \left(\frac{1 - \beta^4}{1 - \beta^4 \tau^{2/\kappa}} \right) \left(\frac{1 - \tau^{(\kappa-1)/\kappa}}{1 - \tau} \right) \right]^{1/2} \quad \text{(venturi tubes)} \quad (2)$$

$$\varepsilon = 1 - (0.41 + 0.35\beta^4) \frac{\Delta p}{\kappa p_1} \quad \text{(orifice plates)} \quad (3)$$

There is currently a draft revision of ISO 5167 containing a revision of Eq. (3) by Reader-Harris [2].

The flow through the V-Cone was initially assumed to be adiabatic, similar to the Venturi meter. However, in 1994, Dahlstrom [3] presented the following empirical equation for the V-Cone expansibility factor based on only two V-Cone meters:

$$\varepsilon = 1 - (0.6 + 0.75\beta^4) \frac{\Delta p}{\kappa p_1} \quad (4)$$

Consequently, it was decided that a more detailed examination to determine the expansibility factor for the V-Cone should be undertaken.

McCrometer asked NEL to perform a series of tests to determine the expansibility for a standard V-Cone meter and on an alternative design of the V-Cone meter, known as the Wafer-Cone. The principal difference is that, unlike the arrangement in the standard V-Cone, the wafer cone is not fixed into the meter, but is inserted as a separate section, enabling removal and replacement with a different beta value, as required, for changing flow conditions.

Owing to the physical difference between the two designs, it was deemed necessary to determine whether or not the expansibility factor of the Wafer-Cone was the same as that of the standard V-Cone.

NEL were chosen to carry out this work and to derive the applicable equation. This paper details the experimental process, the results obtained, and the final derived expansibility equation for the standard V-Cone (4) and for the Wafer-Cone.

2 OBJECTIVES

The key objectives of these tests were as follows:

1. To determine the expansibility factor equation for the standard V-Cone meter.
2. To investigate any dependence of the expansibility factor on pipe diameter.
3. To investigate any dependence of the expansibility factor on Reynolds number.
4. To determine the expansibility factor for the Wafer-Cone meter.

3 EXPERIMENTAL METHOD

To determine the expansibility factor it is necessary to carry out tests at constant Reynolds number (i.e. constant mass flowrate) whilst varying the static pressure and differential pressure at the meter. In order to achieve this constant flowrate a sonic nozzle was used as the reference meter upstream of the V-Cone. The static pressure and differential pressure at the V-Cone were varied using a valve downstream of the V-Cone.

3.1 Standard V-Cone

To achieve the objectives stated above, the following tests were carried out:

- Three 6" V-Cones ($\beta = 0.75, 0.55, \text{ and } 0.45$) were tested at 1 kg/s, giving an approximate pipe Reynolds number, $Re_{pipe} \approx 0.5 * 10^6$. These are test numbers 1, 2, and 3.
- Next, one of these 6" V-Cones ($\beta = 0.45$) was tested at a second flowrate of 0.5 kg/s, giving a pipe Reynolds number, $Re_{pipe} \approx 0.25 * 10^6$. This is test number 4.
- One 4" V-Cone ($\beta = 0.55$) was tested at flowrates of 0.33 kg/s and 0.66 kg/s, to give the same approximate pipe Reynolds number as with the 6" tests. These are test numbers 5 and 6.

On completion of these tests it was felt that there was insufficient data from which to derive an equation for expansibility in a V-Cone, particularly at high beta values, with only one data set with $\beta > 0.55$. Subsequently, it was decided to carry out the following additional tests:

- Another 4" V-Cone ($\beta = 0.65$) was tested at flowrates of 0.33 kg/s and 0.66 kg/s, to give the same approximate pipe Reynolds numbers as with the 6" tests. These are test numbers 7 and 8.
- A 3" V-Cone ($\beta = 0.75$) was tested at a flowrate of 0.5 kg/s, again to give the same approximate pipe Reynolds number as with the 6" tests. This is test number 9.

Details of the V-Cones and tests are summarised in Table 1.

Table 1 - Details of standard V-Cones and tests

<i>Test No.</i>	<i>Size (inch)</i>	<i>β (-)</i>	<i>Flowrate (kg/s)</i>
1	6	0.75	1
2	6	0.55	1
3	6	0.45	1
4	6	0.45	0.5
5	4	0.55	0.66
6	4	0.55	0.33
7	4	0.65	0.66
8	4	0.65	0.33
9	3	0.75	0.5

The V-Cones were installed one at a time in the NEL High Pressure Gravimetric Rig in the Gas Flow Lab with 3", 4" or 6" pipework upstream and downstream of the V-Cone as appropriate to the meter size. The reference flowrate was measured using a sonic nozzle upstream of the V-Cone. The temperature and pressure were measured upstream of the sonic nozzle. The static pressure upstream of the V-Cone, and the differential pressure across it, were measured, as were the temperature downstream of the V-Cone and the barometric pressure. A control valve was located some distance downstream of the V-Cone to enable the static/differential pressure at the V-Cone to be varied.

3.2 Wafer-Cone

A similar test procedure was used for the Wafer-Cone. The Wafer-Cones selected for testing were a set of four 4-inch meters, with beta values of 0.45, 0.55, 0.65, and 0.75. This selection was based on a desire to cover the same beta range as in the original tests with the standard V-Cone.

The original tests on the standard V-Cone had shown that the expansibility factor was not dependent on pipe Reynolds number, and hence mass flowrate. The flowrate for the Wafer-Cone was therefore chosen to give sufficiently large Δp values for each Wafer-Cone, but not so large as to be unrepresentative of the conditions likely to be encountered in industrial application.

Accordingly the Wafer-Cones with beta ratios of 0.45, 0.55, and 0.65 were tested at approximately 0.5 kg/s, giving an approximate pipe Reynolds number of 3.8×10^5 , and the beta 0.75 Wafer-Cone was tested at approximately 0.68 kg/s, giving an approximate pipe Reynolds number of 5.1×10^5 . Details of the Wafer-Cone tests are summarised in Table 2 below.

Table 2 - Details of Wafer-Cones and tests

<i>Test No.</i>	<i>Size (inch)</i>	<i>β (-)</i>	<i>Flowrate (kg/s)</i>
1	4	0.45	0.52
2	4	0.55	0.52
3	4	0.65	0.52
4	4	0.75	0.67

4 DISCUSSION OF RESULTS

4.1 Standard V-Cone

In order to discuss the results in a satisfactory manner it is first necessary to describe the method used for determining the expansibility factor from the data collected.

4.1.1 Method of determining expansibility factor

From the flow equation for a differential pressure device, Eq. (1), it can be seen that there are two unknowns from our test results, the discharge coefficient, C_d , and the expansibility factor, ε . Eq. (1) can be rearranged to give:

$$C_d \varepsilon = \frac{q_m \sqrt{1 - \beta^4}}{\frac{\pi}{4} d^2 \sqrt{2 \Delta p \rho_1}} \quad (5)$$

The functional form of the equation for V-Cone expansibility factor must be decided before proceeding further with the analysis. If Eq. (2) is expanded for small $\Delta p / p_1$ and small β^4 , then it, Eq. (3), and the revised orifice plate expansibility factor equation in ISO/DIS 5167-2 [2] are all of the form:

$$\varepsilon = 1 - (a + b\beta^4) \frac{\Delta p}{\kappa p_1} \quad (6)$$

Accordingly, Figures A.1 to A.9 (in Appendix A) show the experimental values of $C_d \varepsilon$ plotted against $\frac{\Delta p}{\kappa p_1}$. The resultant plot has a reasonably linear form, as can be seen in all the graphs.

It can be seen from Eq. (6) that as $\frac{\Delta p}{\kappa p_1}$ tends to zero, the expansibility factor tends to unity.

Therefore, if a best fit linear curve is calculated through the data in the form $y = mx + c$, then the intercept c can be taken as the discharge coefficient C_d as ε will be unity.

From the slope m , the term in brackets in Eq. (6) can be determined. In order to evaluate this term in brackets, i.e. constant and dependence on β , there needs to be data over a wide enough range of β values. After the additional two meters were tested, data was available for beta values of 0.45, 0.55, 0.65, and 0.75. This is sufficient to be able to develop a reliable equation for the expansibility factor.

4.1.2 Analysis of results

Using the method described above, the resultant values for m and c are given in Table 3 below.

Table 3. Linear fits through data from standard V-Cone tests 1 to 9.

Test No.	Size (inch)	β	Flow (kg/s)	m	c
1	6	0.75	1	-0.6899	0.8255
2	6	0.55	1	-0.6275	0.8800
3	6	0.45	1	-0.6291	0.8778
4	6	0.45	0.5	-0.5599	0.8695
5	4	0.55	0.66	-0.5488	0.8361
6	4	0.55	0.33	-0.5808	0.8316
7	4	0.65	0.66	-0.6615	0.8306
8	4	0.65	0.33	-0.6981	0.8232

9	3	0.75	0.5	-0.6994	0.8085
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The values of m and c give the linear fit through the $C_d \varepsilon$ against $\Delta p / \kappa p_1$ data for each test. The parameter c is the measured value of C_d , obtained by extrapolating the linear fit to the y-axis, where ε is unity. Dividing the m values by this C_d value gives the slope of the equation for the expansibility factor, ε . These slopes for each test are given below in Table 4:

Table 4. Slopes of derived expansibility equation for each test.

Test No.	Size (inch)	β	Flow (kg/s)	slope
1	6	0.75	1	-0.8357
2	6	0.55	1	-0.7131
3	6	0.45	1	-0.7167
4	6	0.45	0.5	-0.6439
5	4	0.55	0.66	-0.6564
6	4	0.55	0.33	-0.6984
7	4	0.65	0.66	-0.7964
8	4	0.65	0.33	-0.8480
9	3	0.75	0.5	-0.8651

The final expansibility equation must be derived from the results in Table 4 above. To achieve this, the slopes of the data from each test are shown below in Figure 1 plotted against β^4 .

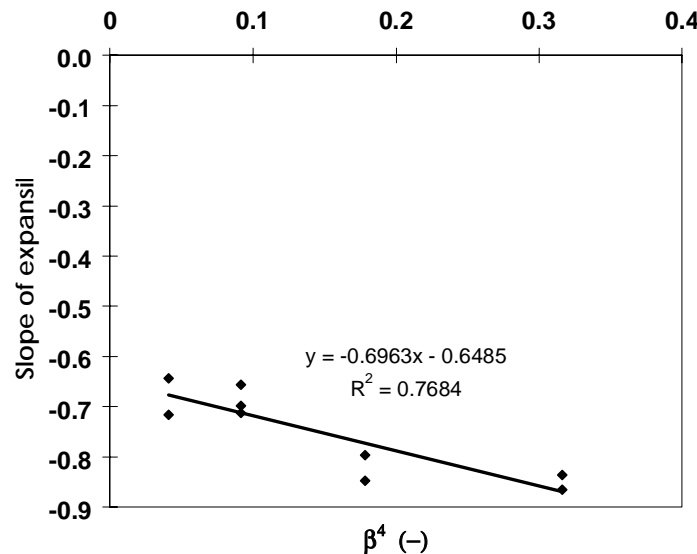


Figure 1. Variation of expansibility slope of standard V-Cone with β^4 .

Altering the power of β yielded no significant improvement in the fit of the slopes from each test. By taking the best-fit line through the slopes from each test, the overall expansibility equation can be determined. This best-fit line is shown in Figure 1. Therefore the resultant equation for the expansibility factor in a standard V-Cone meter is:

$$\varepsilon = 1 - (0.649 + 0.696\beta^4) \frac{\Delta p}{\kappa p_1} \quad (7)$$

4.1.3 Effect of Reynolds number and pipe diameter

As described in Section 2, two key objectives of the tests were to show that the expansibility factor was not significantly affected by pipe diameter or by Reynolds number.

It can be seen from Figure 1 that this is the case. The two values for $\beta = 0.75$ ($\beta^4 = 0.32$) are in good agreement, and come from a 6" V-Cone at 1 kg/s and a 3" V-Cone at 0.5 kg/s, both these tests giving a Reynolds number of approximately 0.5×10^6 . Tests 2 and 5 were undertaken at the same Reynolds number for $\beta = 0.55$ ($\beta^4 = 0.09$) but with 6" and 4" V-Cone respectively, and the trend as to which pipe diameter gives the higher value for the slope is the opposite to that for $\beta = 0.75$.

Equally there are three cases of the same meter having been tested at two different flowrates, and hence Reynolds numbers. Tests 3/4, 5/6, and 7/8 show reasonable agreement within their respective pairs and there is no trend as to which Reynolds number gives the higher value for the slope.

This confirms the fact that the expansibility factor of the standard V-Cone is not affected by the pipe diameter or the Reynolds number.

4.1.4 Comparison with Dahlstrom's equation

As stated in the introduction, Dahlstrom [3] presented Eq. (4) as the expansibility factor for the standard V-Cone meter. Dahlstrom's equation was derived from only five points from two meters. Figures A.10 to A.18 show the test results from the standard V-Cones along with the derived equation for expansibility, Eq. (6), compared with Dahlstrom's equation, Eq. (4). The orifice plate and Venturi tube equations are also shown on the graph as a guide to how the V-Cone expansibility compares with both these devices.

It can be seen that Dahlstrom's equation is reasonably close to the new equation, Eq. (6), for many of the tests. However, the number of points and different meters used in the derivation of the new equation give rise to increased confidence in it. It is also evident that the V-Cone expansibility is somewhat different from that of the Venturi tube, and indeed lies in between the orifice plate and Venturi tube equations, slightly closer to the Venturi.

4.2 Wafer-Cone

The four Wafer-Cones were tested in the same manner as the standard V-Cones. The same method (described in Section 4.1.1) was used to determine the Wafer-Cone expansibility factor. The test results are shown in Figures B.1 to B.4 in Appendix B.

4.2.1 Analysis of results

Using the method described above, the resulting values for m and c are given in Table 5 below.

Table 5. Linear fits through data from Wafer-Cone tests 1 to 4.

Test No.	Size (inch)	β	Nominal Flow (kg/s)	m	c
1	4	0.45	0.52	-0.6825	0.8888
2	4	0.55	0.52	-0.7228	0.8962
3	4	0.65	0.52	-0.8946	0.9169
4	4	0.75	0.67	-1.3070	0.9119

The values of m and c give the linear fit through the $C_d \varepsilon$ against $\Delta p / \rho p_1$ data for each test. The parameter c is the measured value of C_d , obtained by extrapolating the linear fit to the y-axis, where ε is unity. Dividing the m values by this C_d value gives us the slope of the equation for the expansibility factor, ε . These slopes for each test are given below in Table 6.

Table 6. Slopes of derived expansibility equation for each test.

Test No.	Size (inch)	β	Nominal Flow (kg/s)	Slope
1	4	0.45	0.52	0.7679
2	4	0.55	0.52	0.8065
3	4	0.65	0.52	0.9757
4	4	0.75	0.67	1.4333

The final expansibility equation must be derived from the results in Table 6 above. To achieve this, the slopes of the data from each test are plotted against β raised to some power. In the case of the standard V-Cone – and the orifice plate expansibility in ISO 5167-1:1991 – this power is 4. Figure 2 shows the four slopes plotted against β^4 . It can be seen that there is a distinct curve with increasing β^4 , and that the linear fit does not represent this curve.

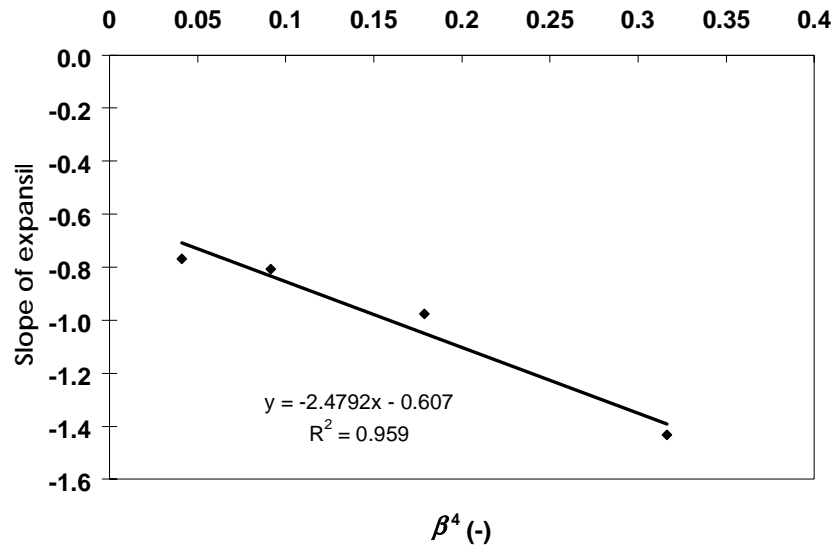


Figure 2. Variation of expansibility slope of Wafer-Cone with β^4 .

To try to improve the fit of the equation, different powers of β were used. Indeed, it was found that by plotting the slopes against β^8 the resulting linear fit was near perfect, as shown in Figure 3.

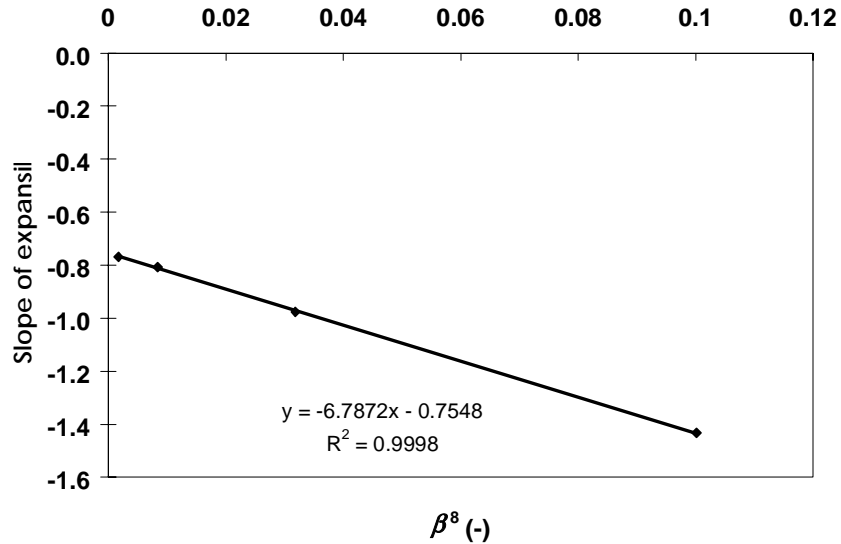


Figure 3. Variation of expansibility slope of Wafer-Cone with β^8 .

By taking the best-fit line through the slopes from each test, the overall expansibility equation can be determined. This best-fit line is shown in Figure 3. Therefore the resulting equation for the expansibility factor in a Wafer-Cone meter, based on these four tests, can be taken as:

$$\varepsilon = 1 - \left(0.755 + 6.787\beta^8\right) \frac{\Delta p}{\kappa p_1} \quad (8)$$

4.2.2 Comparison with standard V-Cone, orifice plate and Venturi

It has been shown that the expansibility factor standard V-Cone lies between that of the orifice plate and the Venturi meters, slightly closer to the Venturi. Figures B.5 to B.8 (Appendix B) show the test results from the Wafer-Cone work compared against the derived equation, Eq. (8), and the standard V-Cone expansibility equation, Eq. (7). The orifice plate and Venturi expansibility equation are also shown for comparison.

It can be seen that the expansibility factor for the Wafer-Cone is significantly different from that of the standard V-Cone. The slope of the expansibility factor for the Wafer-Cone is steeper than that for the standard V-Cone, and lies much closer to that for the Venturi meter. The slope from the $\beta = 0.45$ test is less than that for the Venturi, whilst the slope from the $\beta = 0.55$ test is much closer to the Venturi. The slope from the $\beta = 0.65$ test is actually slightly higher than that for the Venturi equation, and the slope from the $\beta = 0.75$ test is significantly higher than that for the Venturi. It is possible that this is due to the Wafer-Cone having wall tappings for both pressure measurements.

5 CONCLUSIONS

Six standard V-Cone flowmeters were tested in air at NEL's Flow Centre in order to determine the expansibility factor of a gas flowing through a V-Cone.

By testing each meter at a nominally constant flowrate and by varying the static pressure and differential pressure at the meter, it was possible to obtain the necessary data to determine the recommended equation for the expansibility factor, Eq. (7).

In addition, no significant effect on the expansibility factor could be attributed to a change in Reynolds number or pipe diameter.

In addition, tests on four Wafer-Cones have shown that the expansibility factor for the Wafer-Cone is not the same as that for the standard V-Cone, and is closer to that for the Venturi, possibly due to the Wafer-Cone having wall tappings. The slope of the Wafer-Cone expansibility factor is much more dependent on β and is in fact higher than the slope for the Venturi at $\beta = 0.65$ and $\beta = 0.75$.

6. LIST OF NOMENCLATURE

a, b Coefficients in Eq. (5)

C_d Discharge coefficient [-]

d (Throat) diameter [m]

p_1 Static pressure [Pa]

Δp Differential pressure [Pa]

q_m Mass flowrate [kg/s]

Re_{pipe} Pipe Reynolds number, $Re_{pipe} = \frac{4\dot{m}}{\pi d \mu}$ [-]

β Effective diameter ratio [-]

ε Expansibility factor [-]

κ Isentropic exponent [-]

ρ Density [kg/m³]

τ Pressure ratio, $\tau = \frac{p_1 - \Delta p}{p_1}$ [-]

7 REFERENCES

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- [2] International Standards Organisation. Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full, Part 2: Orifice plates. Geneva: International Standards Organisation. ISO/DIS 5167-2, May 2000.
- [3] M.J. Dahlstrom. V-Cone meter: Gas measurement for the real world. North Sea Flow Measurement Workshop, Peebles, Scotland, 1994.
- [4] D.G. Stewart, M.J. Reader-Harris, and R.J.W. Peters. Derivation on an Expansibility Factor for the V-Cone Meter. Flow Measurement 2001, International Conference, Peebles, Scotland, May 2001.

APPENDIX A. DIAGRAMS OF RESULTS FOR THE STANDARD V-CONE

Fig. A.1 - Results from standard V-Cone test 1.

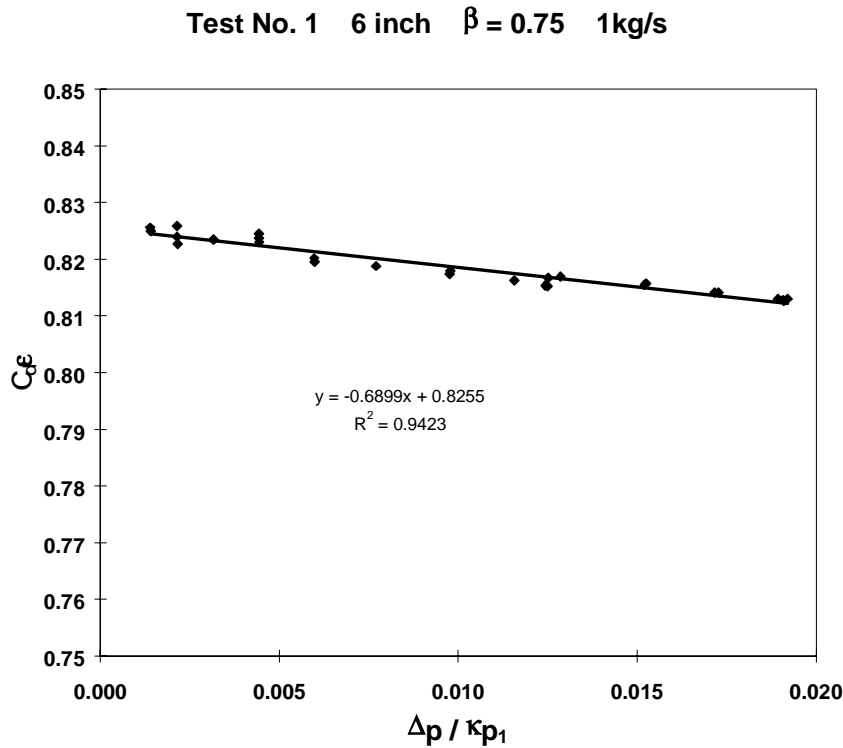


Fig. A.2 - Results from standard V-Cone test 2.

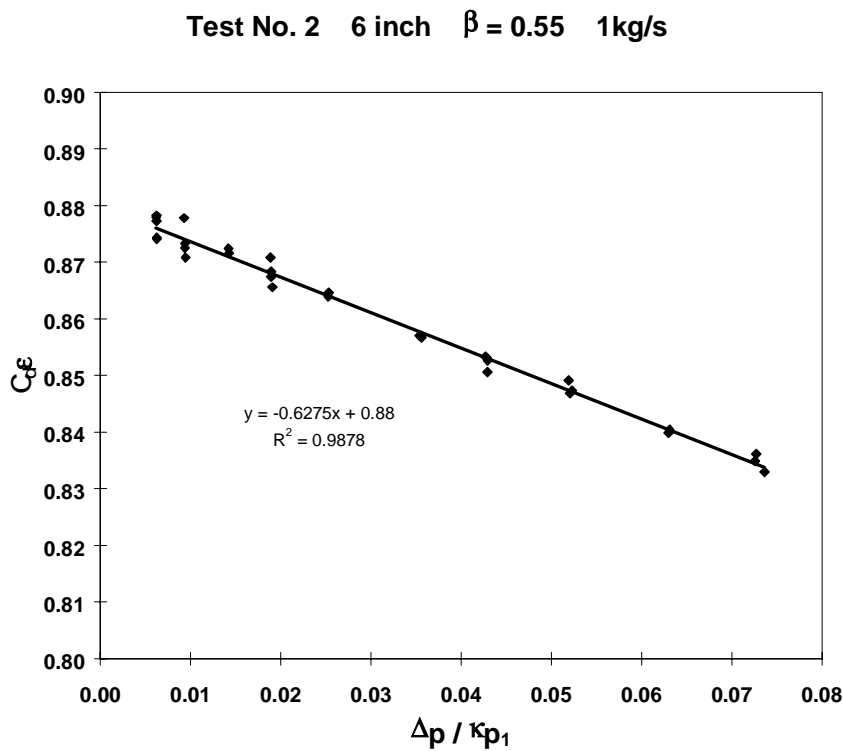


Fig. A.3 - Results from standard V-Cone test 3.

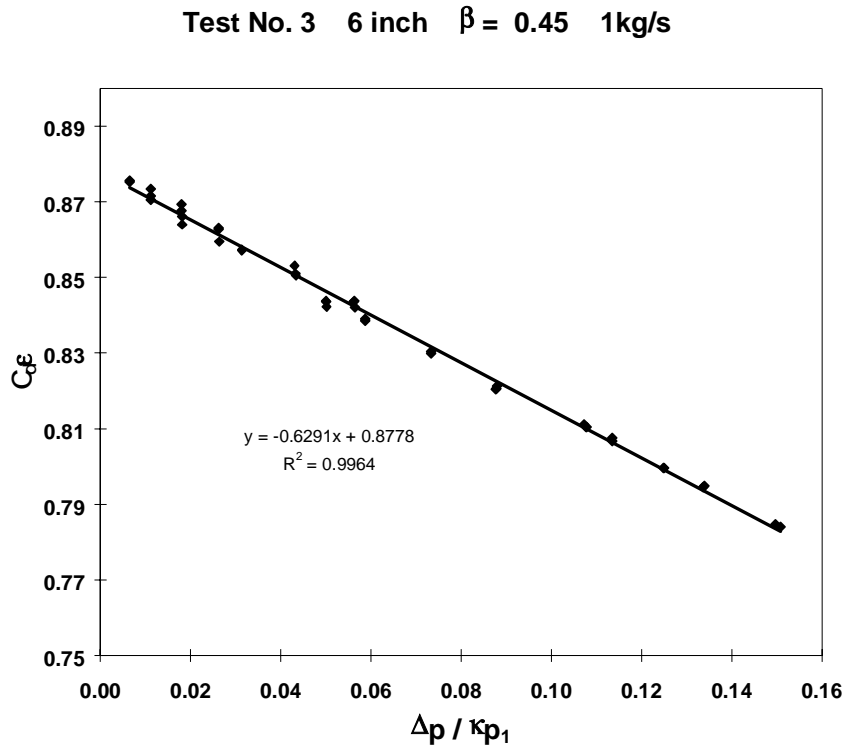


Fig. A.4 - Results from standard V-Cone test 4.

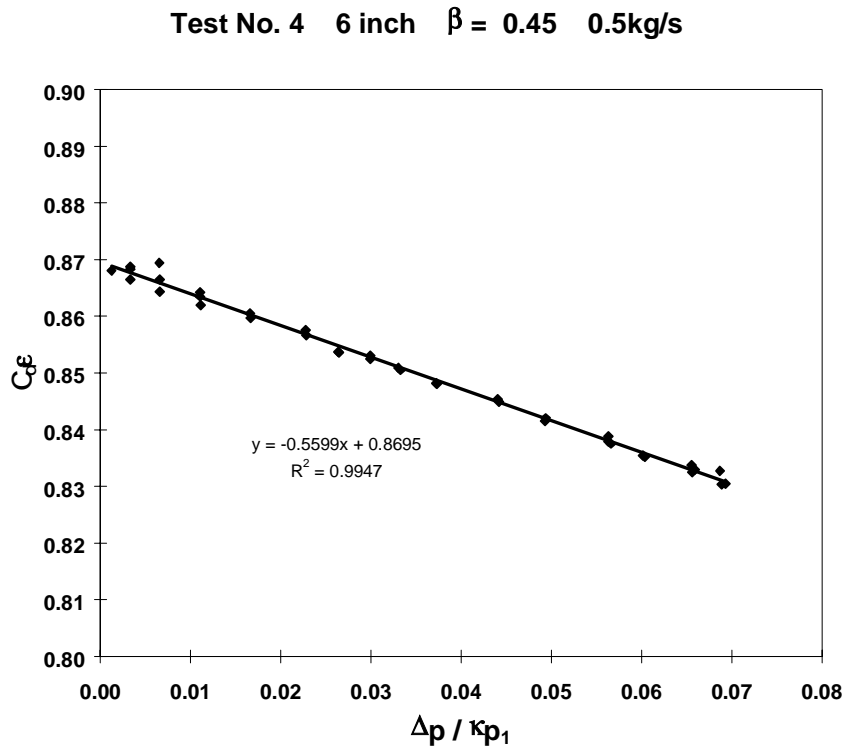


Fig. A.5 - Results from standard V-Cone test 5.

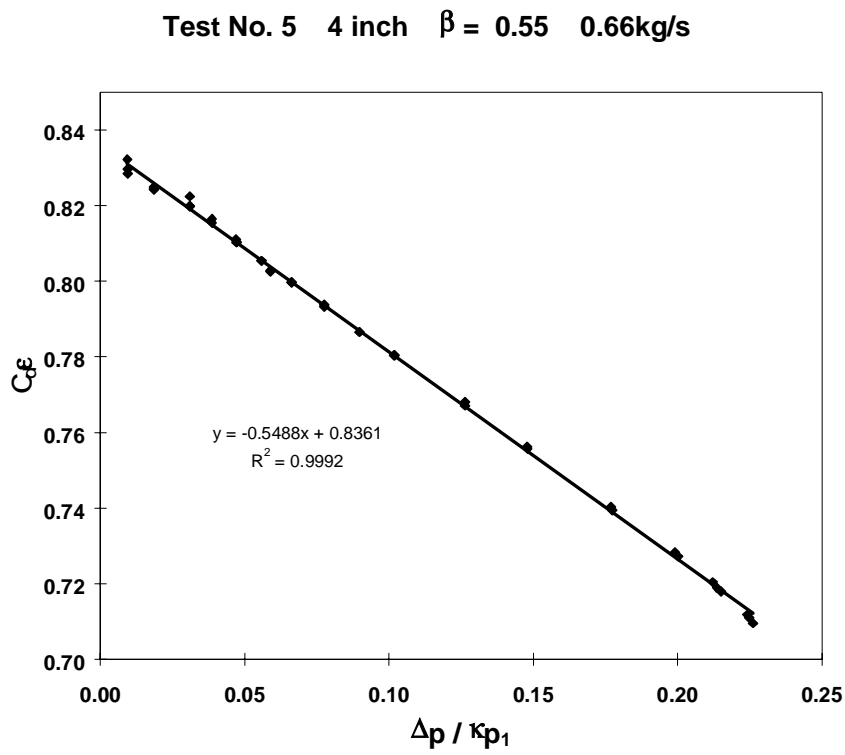


Fig. A.6 - Results from standard V-Cone test 6.

Test No. 6 4 inch $\beta = 0.55$ 0.33kg/s

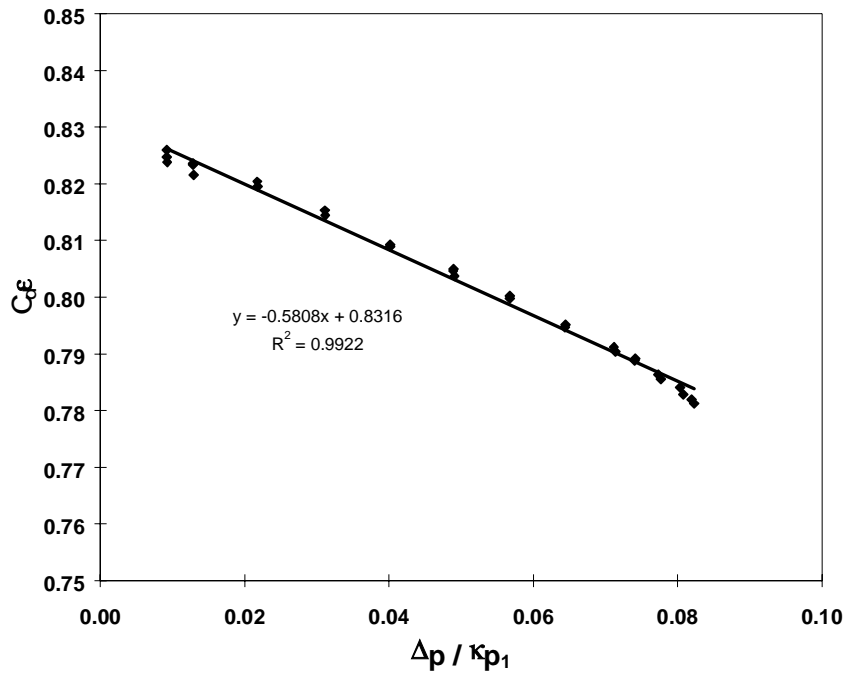


Fig. A.7 - Results from standard V-Cone test 7.

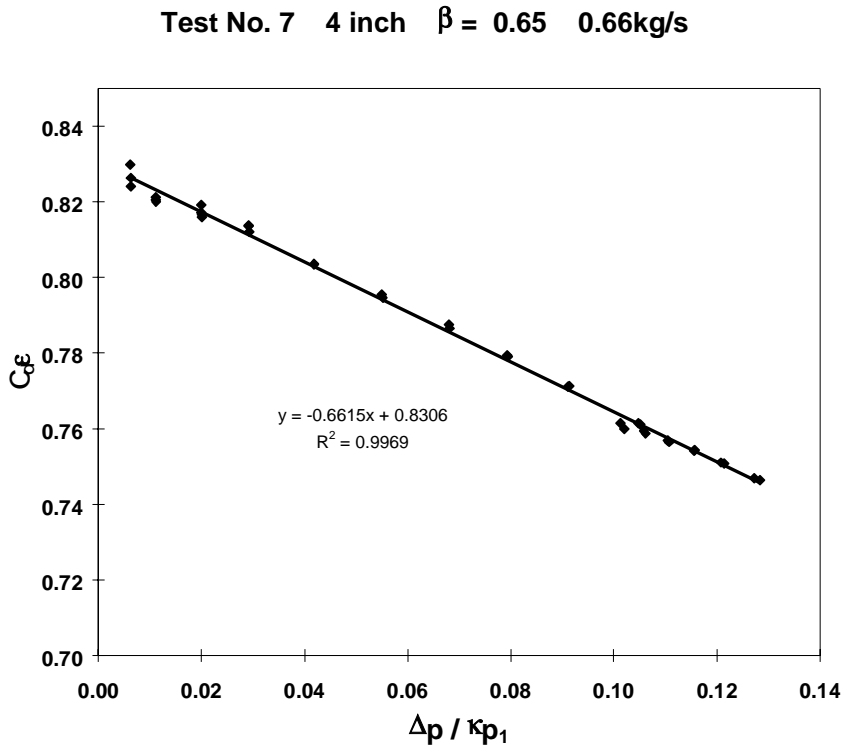


Fig. A.8 - Results from standard V-Cone test 8.

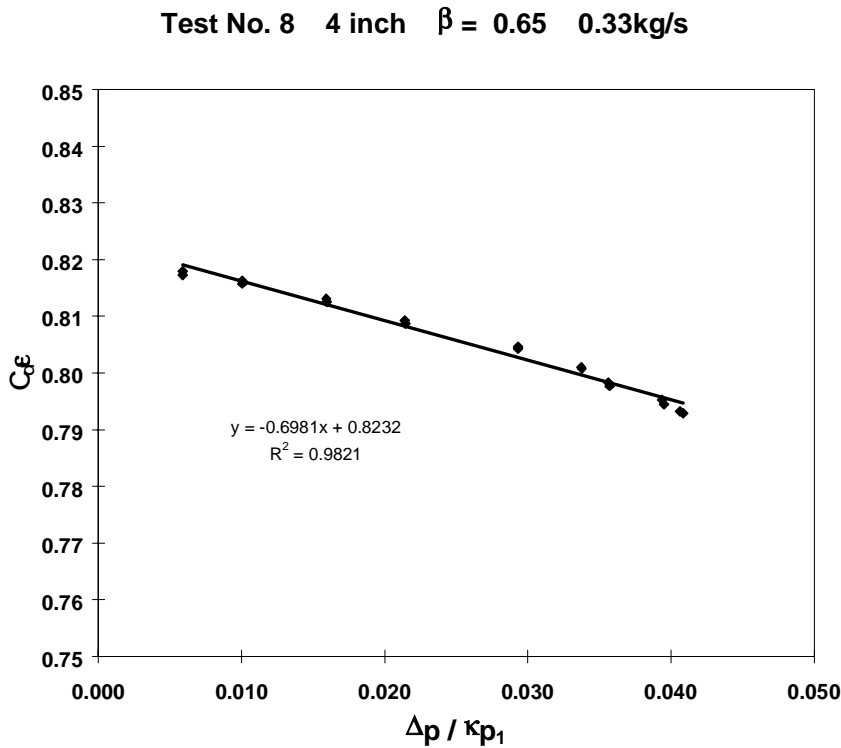


Fig. A.9 - Results from standard V-Cone test 9.

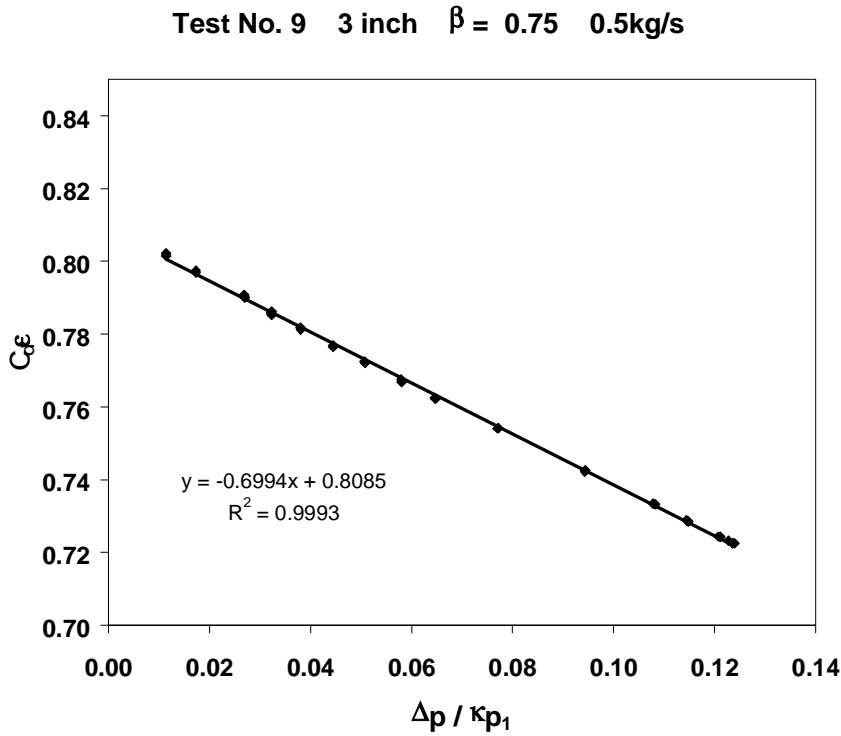


Fig. A.10 - Derived expansibility factor values from standard V-Cone test 1 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

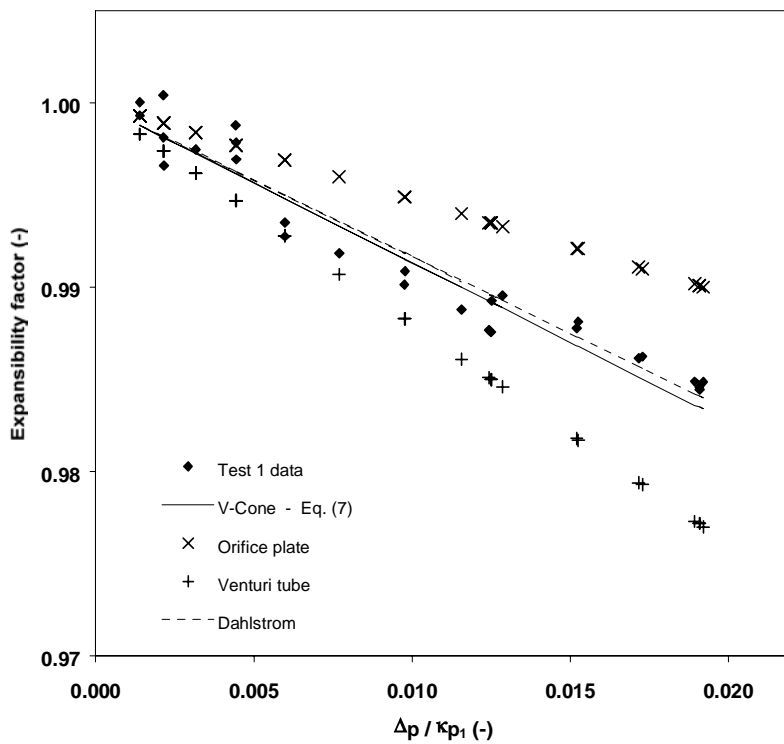


Fig. A.11 - Derived expansibility factor values from standard V-Cone test 2 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

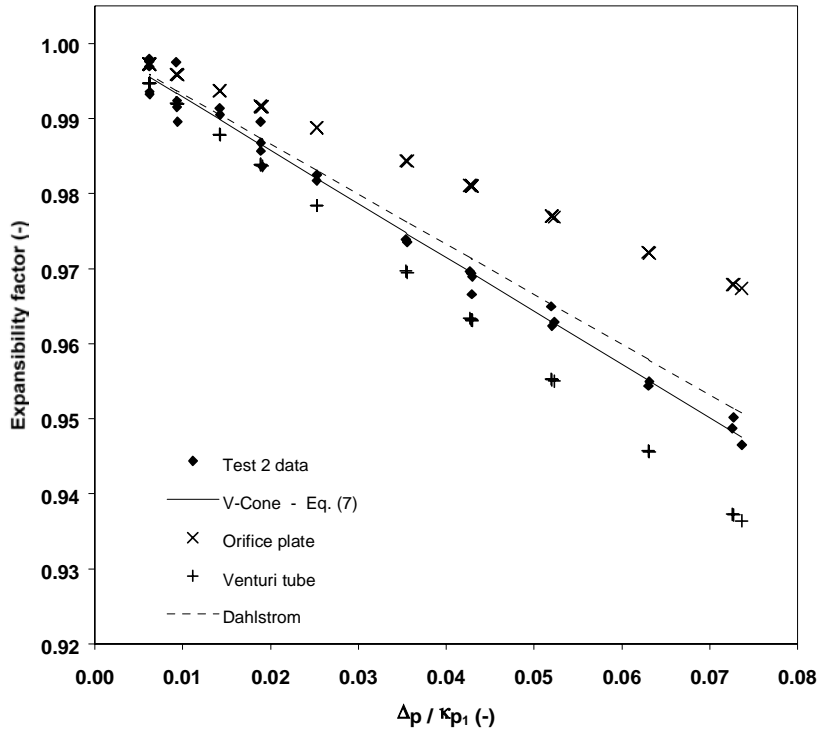


Fig. A.12 - Derived expansibility factor values from standard V-Cone test 3 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

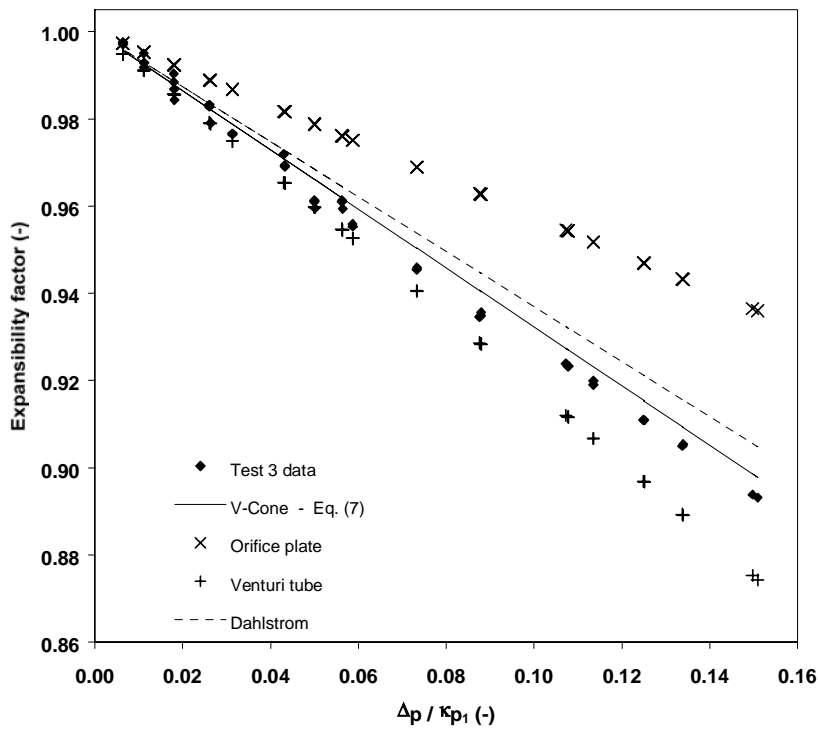


Fig. A.13 - Derived expansibility factor values from standard V-Cone test 4 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

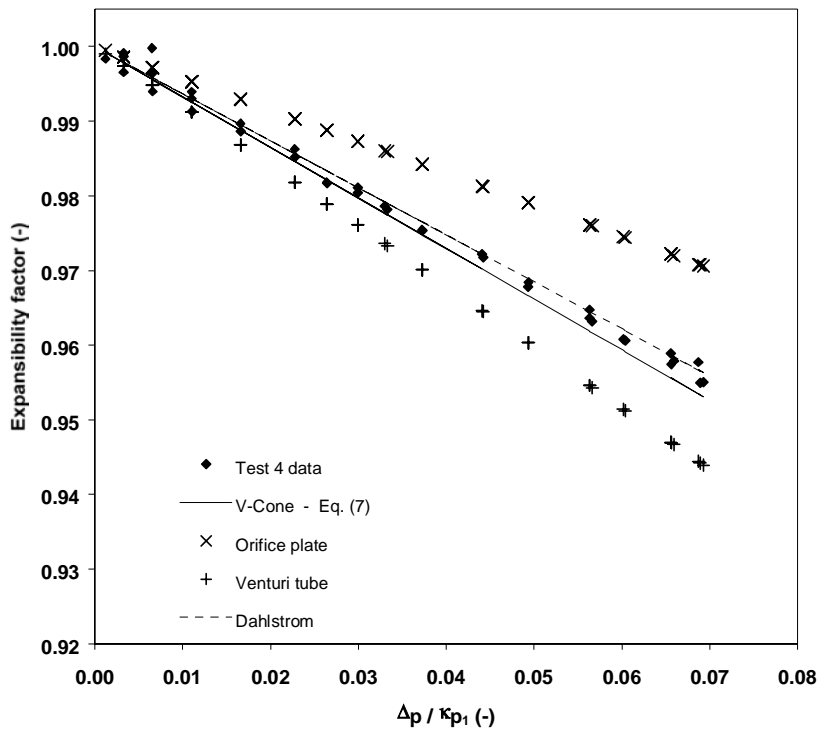


Fig. A.14 - Derived expansibility factor values from standard V-Cone test 5 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

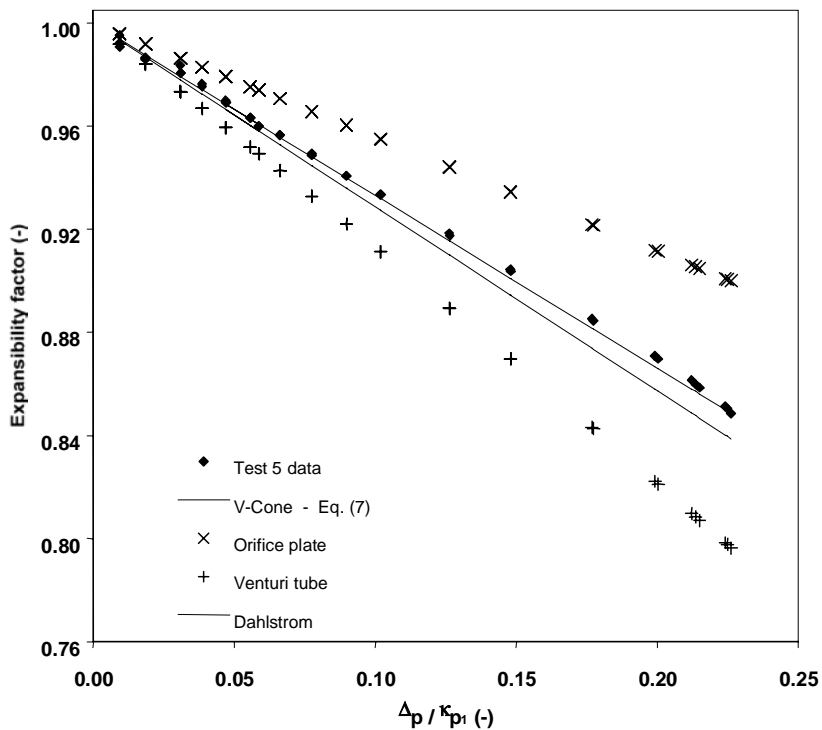


Fig. A.15 - Derived expansibility factor values from standard V-Cone test 6 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

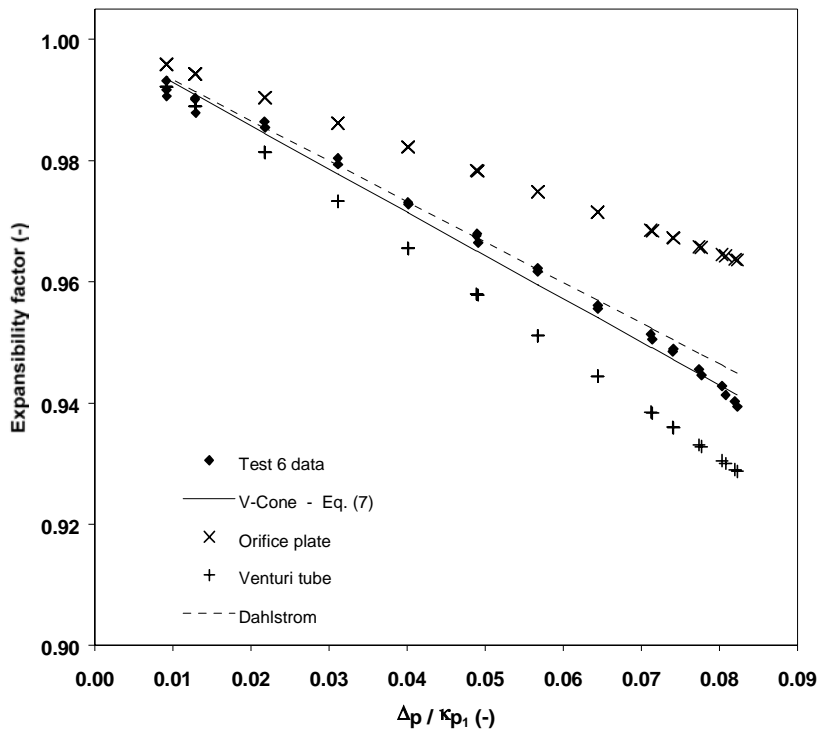


Fig. A.16 - Derived expansibility factor values from standard V-Cone test 7 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

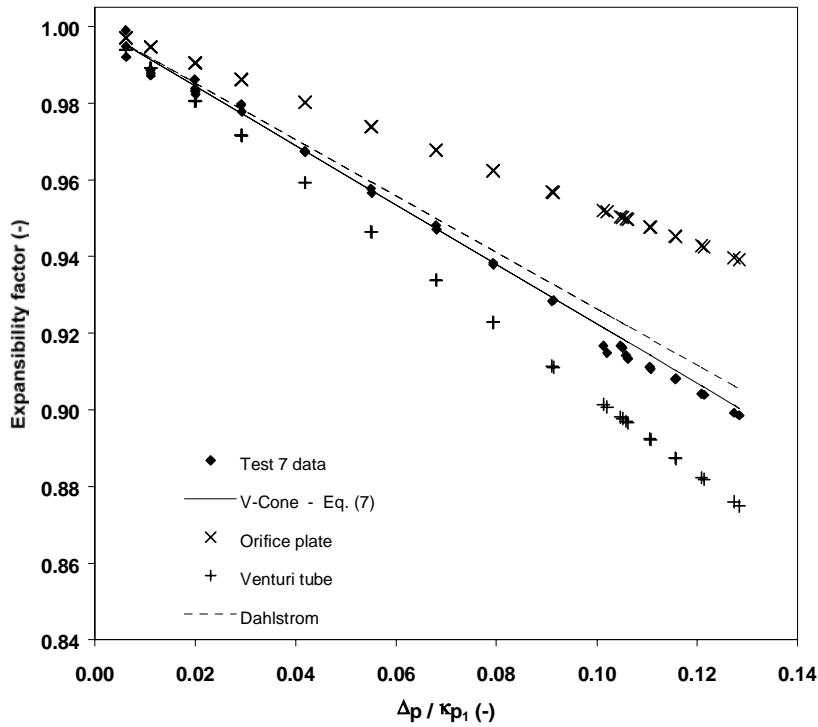


Fig. A.17 - Derived expansibility factor values from standard V-Cone test 8 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

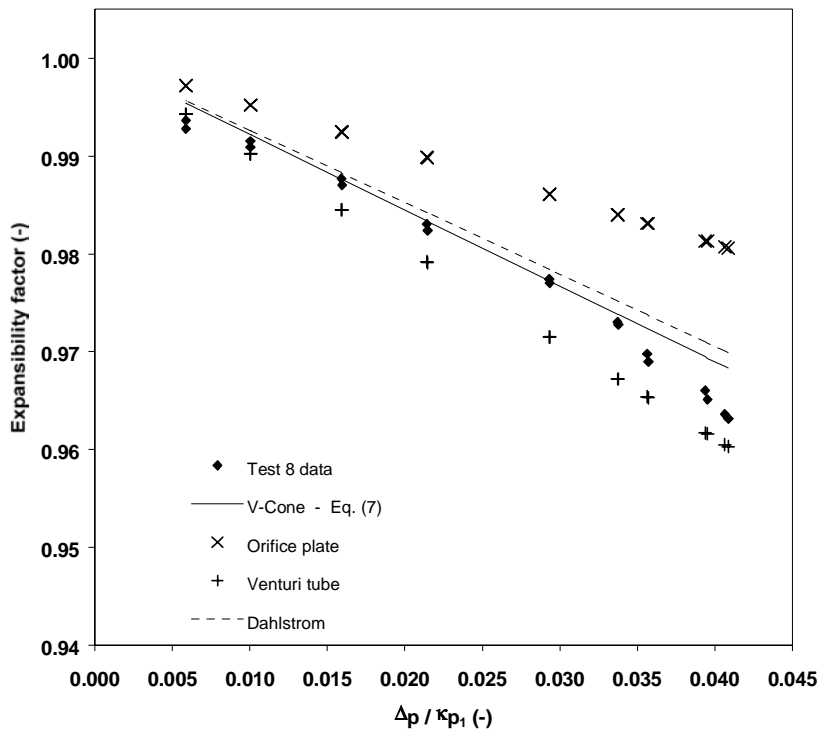
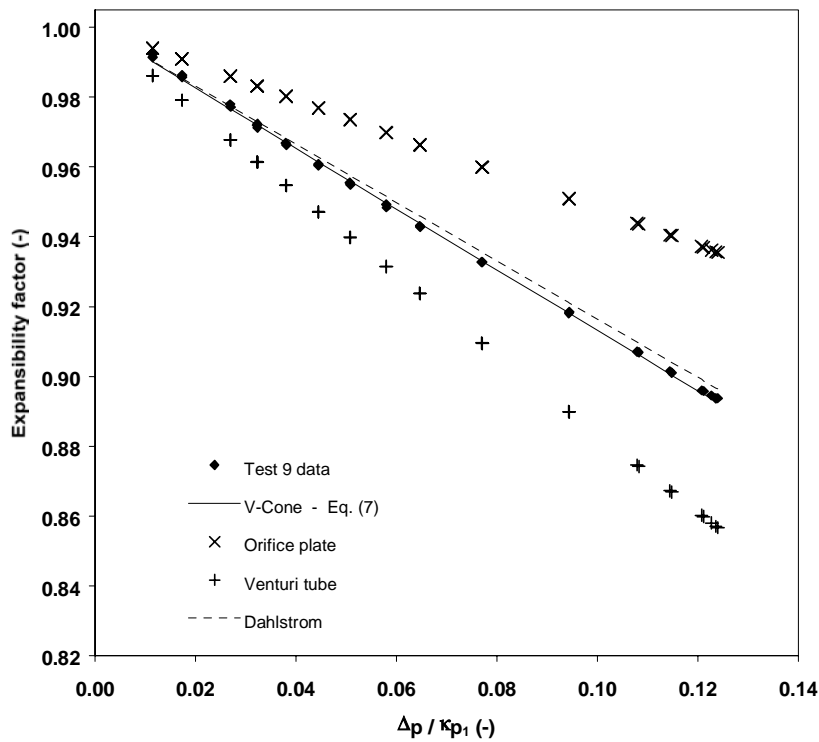


Fig. A.18 - Derived expansibility factor values from standard V-Cone test 9 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.



APPENDIX B. DIAGRAMS OF RESULTS FOR THE WAFER-CONE

Fig. B.1 - $C_d \varepsilon$ against $\Delta p / \kappa p_1$ for Wafer-Cone test 1.

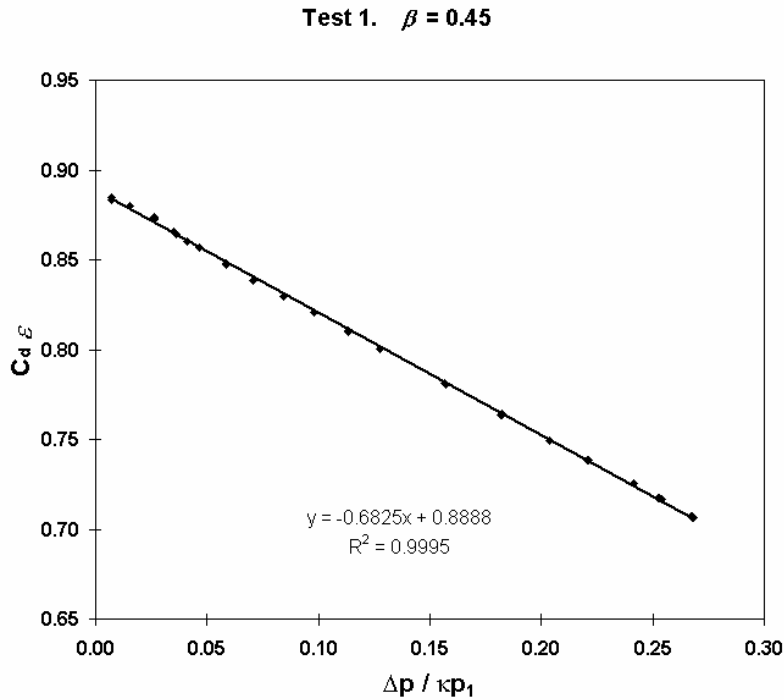


Fig. B.2 - $C_d \varepsilon$ against $\Delta p / \kappa p_1$ for Wafer-Cone test 2.

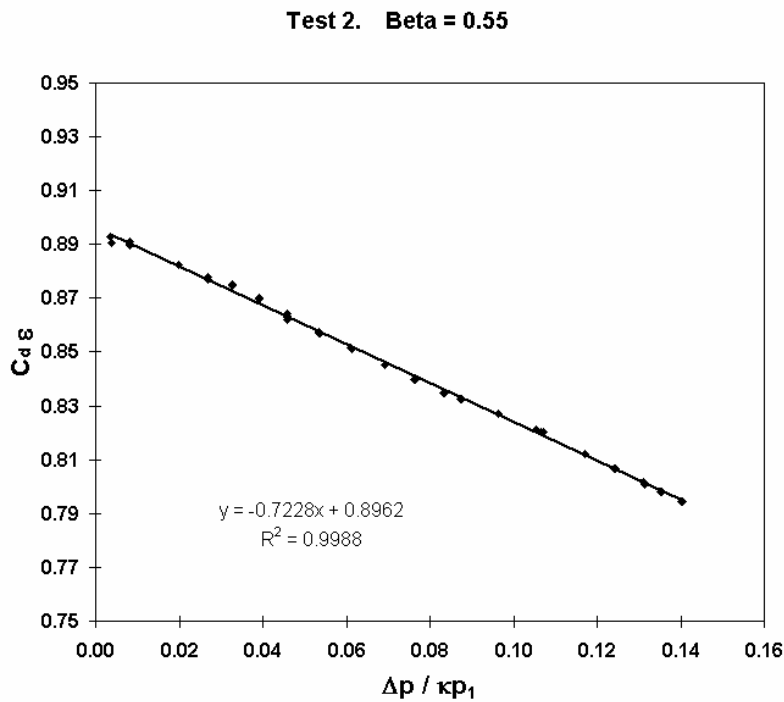


Fig. B.3 - $C_d \varepsilon$ against $\Delta p / \kappa p_1$ for Wafer-Cone test 3.

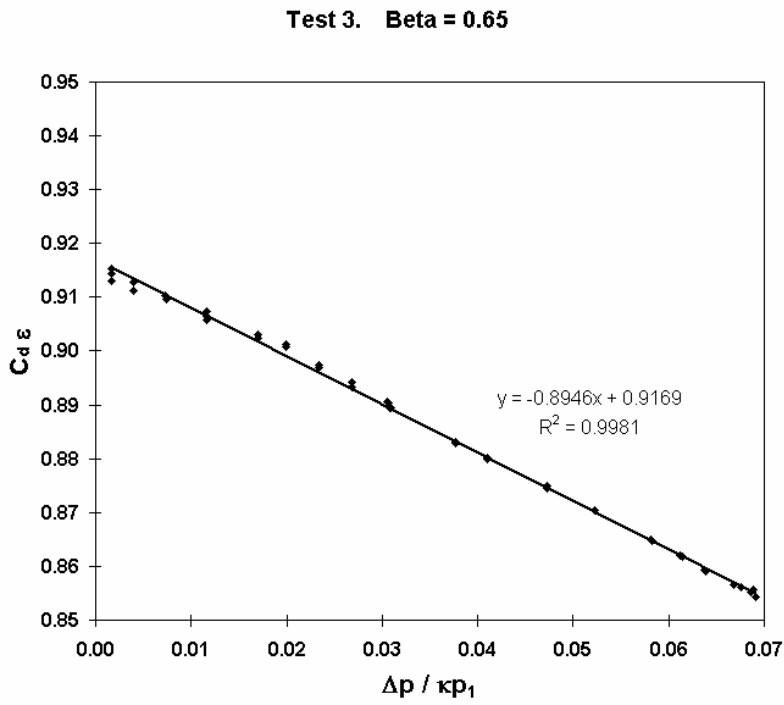


Fig. B.4 - $C_d \varepsilon$ against $\Delta p / \kappa p_1$ for Wafer-Cone test 4.

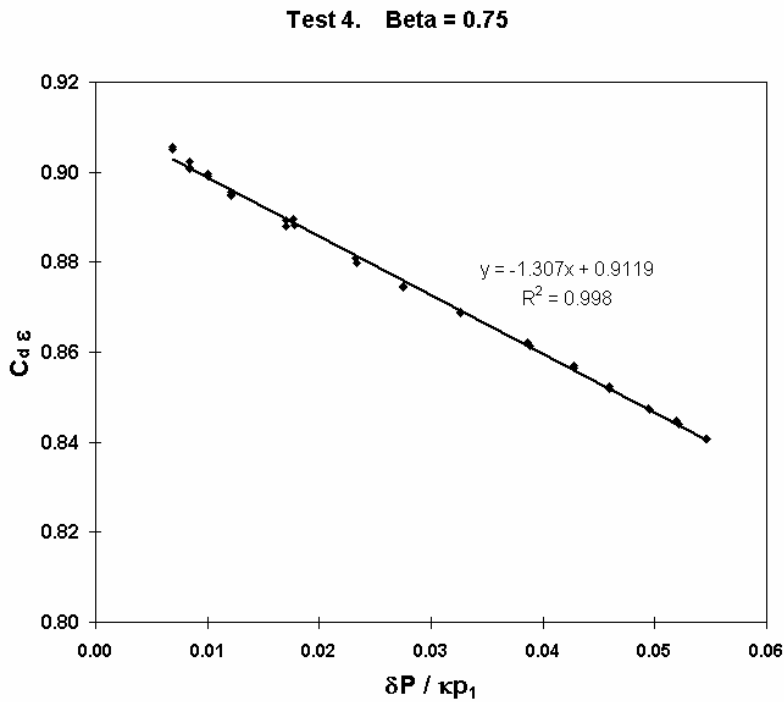


Fig. B.5 - Wafer-Cone results from Test 1 compared with Eq. (8), standard V-Cone expansibility equation, Eq. (7), and orifice plate and Venturi equations.

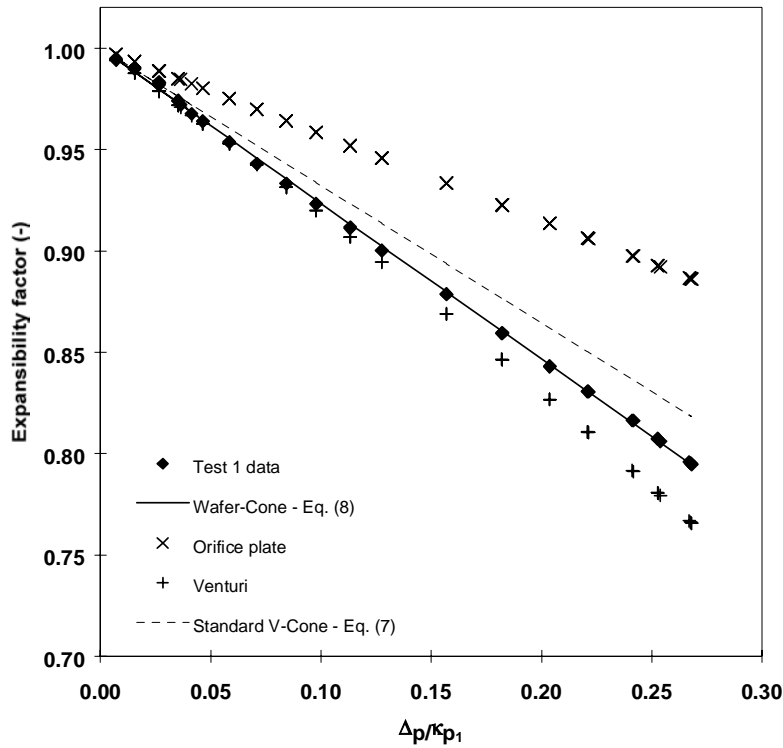


Fig. B.6 - Wafer-Cone results from Test 2 compared with Eq. (8), standard V-Cone expansibility equation, Eq. (7), and orifice plate and Venturi equations.

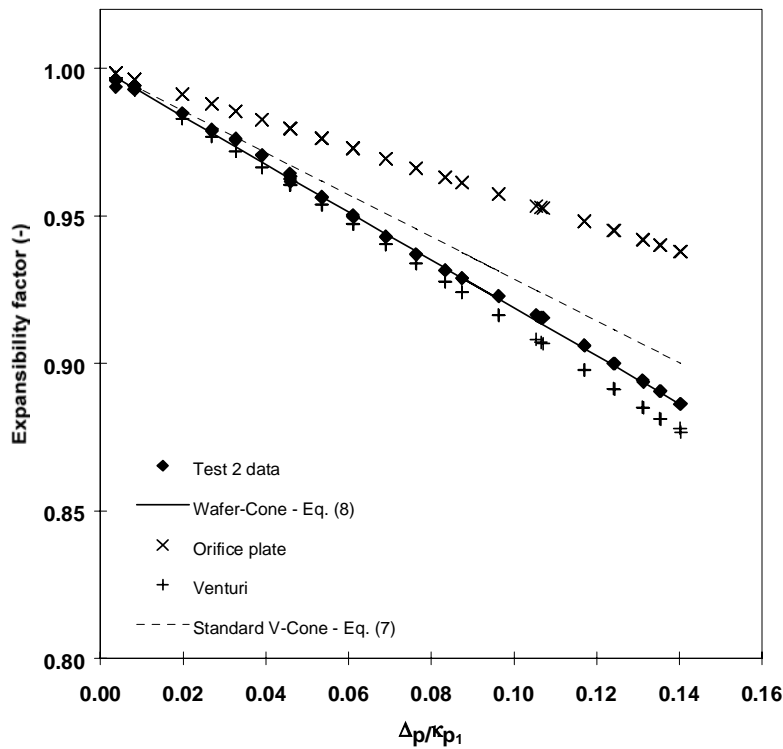


Fig. B.7 - Wafer-Cone results from Test 3 compared with Eq. (8), standard V-Cone expansibility equation, Eq. (7), and orifice plate and Venturi equations.

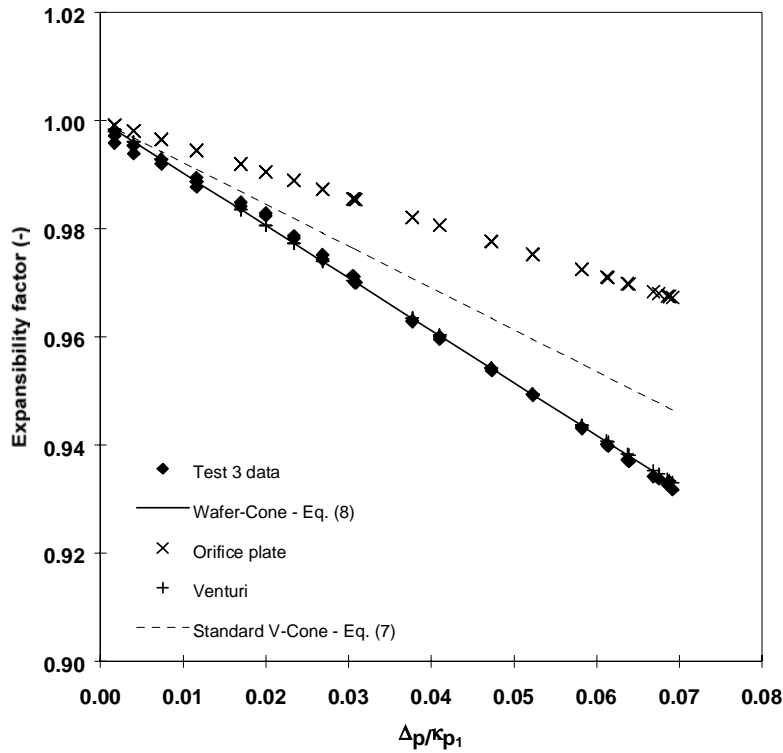


Fig. B.8 - Wafer-Cone results from Test 4 compared with Eq. (8), standard V-Cone expansibility equation, Eq. (7), and orifice plate and Venturi equations.

