DERIVATION OF AN EXPANSIBILITY FACTOR FOR THE V-CONE METER

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1 INTRODUCTION

The V-Cone meter is a differential pressure device that has been developed and tested since 1985 and is now well understood and accepted as a viable flow measurement device. It is widely used in many industrial applications to measure a variety of fluids over a wide range of temperatures and pressures. The choice of the V-Cone can be based on both the short upstream lengths required and the ease of operation over a wide turndown ratio relative to an orifice plate.

Over the past 10 years it has been increasingly accepted in the Gas Industry as a reliable metering device. Due to the fact that gas is a compressible fluid it is necessary to apply an expansibility correction factor, ε (or the *Y* factor in the USA), resulting in the well known mass flow equation:

$$q_m = \frac{C_d}{\sqrt{1-\beta^4}}\varepsilon\frac{\pi}{4}d^2\sqrt{2\Delta p\rho} \qquad (1)$$

Kinghorn [1] in his 1986 paper on this subject stated, "The expansibility coefficient, ε, compensates for the fact that changes in the pressure of the gas as it flows through the meter result in changes in its density. For nozzles and Venturi meters this can be computed on the assumption that the flow is adiabatic since it is constrained by the walls of the meter. For orifice plates, however, the three-dimensional expansion which takes place requires an empirical determination of values of the expansibility coefficient".

The expansibility factor equations for Venturi meters/nozzles and orifice plates are given by ISO 5167-1:1991 as:

$$\varepsilon = \left[\left(\frac{\kappa\tau^{2/\kappa}}{\kappa-1}\right)\left(\frac{1-\beta^4}{1-\beta^4\tau^{2/\kappa}}\right)\left(\frac{1-\tau^{(\kappa-1)/\kappa}}{1-\tau}\right)\right]^{1/2} \qquad \text{(Venturi tubes)} \qquad (2)$$

$$\varepsilon = 1-\left(0.41+0.35\beta^4\right)\frac{\Delta p}{\kappa p_1} \qquad \text{(orifice plates)} \qquad (3)$$

There is currently a draft revision of ISO 5167 containing a revision of Eq. (3) by Reader-Harris [2].

The flow through the V-Cone was initially assumed to be adiabatic, similar to the Venturi meter. However, in 1994, Dahlstrom [3] presented the following empirical equation for the V-Cone expansibility factor based on only two V-Cone meters:

$$\varepsilon = 1-\left(0.6+0.75\beta^4\right)\frac{\Delta p}{\kappa p_1} \qquad (4)$$

Consequently, it was decided that a more detailed examination to determine the expansibility factor for the standard V-Cone should be undertaken. NEL were chosen to carry out this work and to derive the applicable equation. This paper details the experimental process, the results obtained, and the final derived expansibility equation for the standard V-Cone.

2 OBJECTIVES

The key objectives of these tests were as follows:

1. To determine the expansibility factor equation for the standard V-Cone meter.
2. To investigate any dependence of the expansibility factor on pipe diameter.
3. To investigate any dependence of the expansibility factor on Reynolds number.

3 EXPERIMENTAL METHOD

To determine the expansibility factor it is necessary to carry out tests at constant mass flowrate (i.e. constant Reynolds number) whilst varying the static pressure and differential pressure at the meter. In order to achieve this constant flowrate a sonic nozzle was used as the reference meter. The static pressure and differential pressure at the V-Cone were varied using a valve downstream of the V-Cone.

To achieve the objectives stated above, the following tests were carried out:

- Three 6" V-Cones (β = 0.75, 0.55, and 0.45) were tested at 1 kg/s, giving an approximate pipe Reynolds number, $Re_D \approx 0.5 * 10^6$. These are test numbers 1, 2, and 3.
- Next, one of these 6" V-Cones (β = 0.45) was tested at a second flowrate of 0.5 kg/s, giving a pipe Reynolds number, $Re_D \approx 0.25 * 10^6$. This is test number 4.
- One 4" V-Cone (β = 0.55) was tested at flowrates of 0.33 kg/s and 0.66 kg/s, to give the same approximate pipe Reynolds number as for the 6" tests. These are test numbers 5 and 6.

On completion of these tests it was felt that there was insufficient data from which to derive an equation for expansibility in a V-Cone, particularly at high beta values, with only one data set with $\beta > 0.55$. Subsequently, it was decided to carry out the following additional tests:

- Another 4" V-Cone (β = 0.65) was tested at flowrates of 0.33 kg/s and 0.66 kg/s, to give the same approximate pipe Reynolds numbers as for the 6" tests. These are test numbers 7 and 8.

- A 3" V-Cone (β = 0.75) was tested at a flowrate of 0.5 kg/s, again to give the same approximate pipe Reynolds number as for the 6" tests. This is test number 9.

Details of the V-Cones and tests are summarised in Table 1.

Table 1 - Details of V-Cones and Tests

Test No.	Meter serial number	Size (in)	β (-)	Flowrate (kg/s)
1	00-1012	6	0.75	1
2	00-1013	6	0.55	1
3	00-1014	6	0.45	1
4	00-1014	6	0.45	0.5
5	00-2128	4	0.55	0.66
6	00-3393	4	0.55	0.33
7	00-3393	4	0.65	0.66
8	00-3393	4	0.65	0.33
9	01-0140	3	0.75	0.5

The V-Cones were installed one at a time in the NEL High Pressure Gravimetric Rig in the Gas Flow Lab with 3", 4" or 6" pipework upstream and downstream of the V-Cone as appropriate to the meter size. The reference flowrate was measured using a sonic nozzle upstream of the V-Cone. The temperature and pressure were measured upstream of the sonic nozzle. The static pressure upstream of the V-Cone, and the differential pressure across it were measured, as were the temperature downstream of the V-Cone and the barometric pressure. A control valve was located some distance downstream of the V-Cone to enable the static/differential pressure at the V-Cone to be varied.

4 DISCUSSION OF RESULTS

In order to discuss the results in a satisfactory manner it is first necessary to describe the method used for determining the expansibility factor from the data collected.

4.1 Method of Determining Expansibility Factor

From the flow equation for a differential pressure device, Eq. (1), it can be seen that there are two unknowns from our test results, the discharge coefficient, C_d, and the expansibility factor, ε. Eq. (1) can be rearranged to give:

$$C_d\,\varepsilon = \frac{q_m\sqrt{1-\beta^4}}{\frac{\pi}{4}d^2\sqrt{2\Delta p\rho_1}} \tag{5}$$

The functional form of the equation for V-Cone expansibility factor must be decided before proceeding further with the analysis. If Eq. (2) is expanded for small $\Delta p/p_1$ and small β^4, then it, Eq. (3), and the revised orifice plate expansibility factor equation in ISO/DIS 5167-2 [2] are all of the form:

$$\varepsilon = 1-\left(a+b\beta^4\right)\frac{\Delta p}{\kappa p_1} \tag{6}$$

Accordingly, Figures A.1 to A.9 (in Appendix A) show the experimental values of $C_d\varepsilon$ plotted against $\frac{\Delta p}{\kappa p_1}$. The resultant plot has a reasonably linear form, as can be seen in all the graphs. It can be seen from Eq. (6) that as $\frac{\Delta p}{\kappa p_1}$ tends to zero, the expansibility factor tends to unity. Therefore, if a best fit linear curve is calculated through the data in the form $y = mx + c$, then the intercept c can be taken as the discharge coefficient C_d as ε will be unity.

From the slope m, the term in brackets in Eq. (6) can be determined. In order to evaluate this term in brackets, i.e. constant and dependence on β, there needs to be data over a wide enough range of β values. After the additional two meters were tested, data was available for β values of 0.45, 0.55, 0.65, and 0.75. This is sufficient to be able to develop a reliable equation for the expansibility factor.

4.2 Analysis of Results

Using the method described above, the resulting values for *m* and *c* are given in Table 2 below.

Table 2. Linear fits through data from tests 1 to 9.

Test No.	Size (inch)	β	Flow (kg/s)	m	c
1	6	0.75	1	-0.6899	0.8255
2	6	0.55	1	-0.6275	0.8800
3	6	0.45	1	-0.6291	0.8778
4	6	0.45	0.5	-0.5599	0.8695
5	4	0.55	0.66	-0.5488	0.8361
6	4	0.55	0.33	-0.5808	0.8316
7	4	0.65	0.66	-0.6615	0.8306
8	4	0.65	0.33	-0.6981	0.8232
9	3	0.75	0.5	-0.6994	0.8085

The values of *m* and *c* give the linear fit through the $C_d\varepsilon$ against $\Delta p/\kappa p_1$ data for each test. The parameter *c* is the measured value of C_d, obtained by extrapolating the linear fit to the y-axis, where ε is unity. Dividing the *m* values by this C_d value gives us the slope of the equation for the expansibility factor, ε. These slopes for each test are given below in Table 3:

Table 3. Slopes of derived expansibility equation for each test.

Test No.	Size (inch)	β	Flow (kg/s)	slope
1	6	0.75	1	-0.8357
2	6	0.55	1	-0.7131
3	6	0.45	1	-0.7167
4	6	0.45	0.5	-0.6439
5	4	0.55	0.66	-0.6564
6	4	0.55	0.33	-0.6984
7	4	0.65	0.66	-0.7964
8	4	0.65	0.33	-0.8480
9	3	0.75	0.5	-0.8651

The final expansibility equation must be derived from the results in Table 3 above. To achieve this, the slopes of the data from each test are shown below in Figure 1 plotted against β^4.

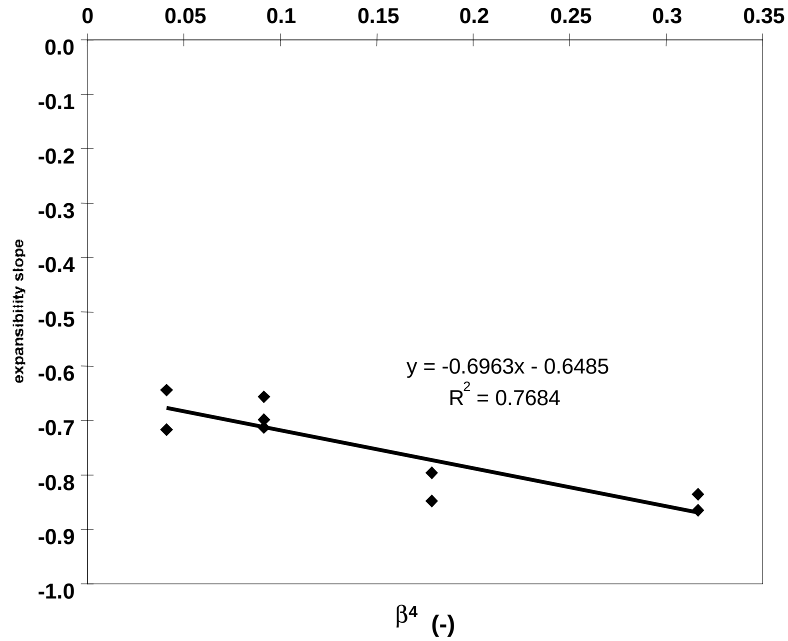

Figure 1. Variation of expansibility slope with β^4.

Altering the power of β yielded no significant improvement in the fit of the slopes from each test. By taking the best-fit line through the slopes from each test, the overall expansibility equation can be determined. This best-fit line is shown in Figure 1. Therefore the resulting equation for the expansibility factor in a V-Cone meter is:

$$\varepsilon = 1 - \left(0.649 + 0.696\beta^4\right)\frac{\Delta p}{\kappa p_1} \tag{7}$$

4.3 Effect of Reynolds Number and Pipe Diameter

As described in Section 2, two key objectives of the tests were to show that the expansibility factor was not significantly affected by pipe diameter or by Reynolds number.

It can be seen from Figure 1 that this is the case. The two values for $\beta = 0.75$ ($\beta^4 = 0.32$) are in good agreement, and come from a 6" V-Cone at 1 kg/s and a 3" V-Cone at 0.5 kg/s, both these tests giving a Reynolds number of approximately 0.5 x 10^6. Tests 2 and 5 were undertaken at the same Reynolds number for $\beta = 0.55$ ($\beta^4 = 0.09$) but with 6" and 4" V-Cone respectively, and the trend as to which pipe diameter gives the highest value for the slope is the opposite to that for $\beta = 0.75$.

Equally there are three cases of the same meter having been tested at two different flowrates, and hence Reynolds numbers. Tests 3/4, 5/6, and 7/8 show reasonable agreement within their respective pairs and there is no trend as to which Reynolds number gives the higher value for the slope.

This confirms that the expansibility factor is not affected by either the pipe diameter or the Reynolds number.

4.4 Comparison with Dahlstrom's Equation

As stated in the introduction, Dahlstrom [3] presented Eq. (4) as the expansibility factor for the standard V-Cone meter. Dahlstrom's equation was derived from only five points from two meters. Figures A.10 to A.18 show the test results from this work along with the derived equation for expansibility, Eq. (6), compared with Dahlstrom's equation, Eq. (4). The orifice plate and Venturi tube equations are also shown on the graph as a guide to how the V-Cone expansibility compares with that from both these devices.

It can be seen that Dahlstrom's equation is reasonably close to the new equation, Eq. (6), for many of the tests. However, the number of points and different meters used in the derivation of the new equation give rise to increased confidence in it. It is also evident that the V-Cone expansibility is somewhat different from that of the Venturi tube, and indeed lies between the orifice plate and Venturi tube equations, slightly closer to the Venturi.

5 CONCLUSIONS

Six V-Cone flowmeters were tested in air at NEL's Flow Centre in order to determine the expansibility factor of a gas flowing through a V-Cone.

By testing each meter at a nominally constant flowrate and by varying the static pressure and differential pressure at the meter, it was possible to obtain the necessary data to determine the recommended equation for the expansibility factor, Eq. (7).

In addition, no significant effect on the expansibility factor could be attributed to a change in Reynolds number or pipe diameter.

6 LIST OF NOMENCLATURE

a, b Coefficients in Eq. (5)

C_d Discharge coefficient [-]

d (Throat) diameter [m]

p_1 Static pressure [Pa]

Δp Differential pressure [Pa]

q_m Mass flowrate [kg/s]

Re_D Pipe reynolds number, $Re_D = \dfrac{4\dot{m}}{\pi\, d\mu}$ [-]

β Effective diameter ratio [-]

ε Expansibility factor [-]

κ Isentropic exponent [-]

ρ Density [kg/m^3]

τ Pressure ratio, $\tau = \dfrac{p_1 - \Delta p}{p_1}$ [-]

7 REFERENCES

[1] F.C. Kinghorn. The expansibility correction for orifice plates: EEC data. In *Proc. Int. Conf. on Flow Measurment in the Mid 80's (Paper 5.2). National Engineering Laboratory East Kilbride, Glasgow, June 1986.*

[2] International Standards Organisation. *Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full, Part 2: Orifice plates. Geneva: International Standards Organisation. ISO/DIS 5167-2, May 2000.*

[3] M.J. Dahlstrom. V-Cone meter: Gas measurement for the real world. *North Sea Flow Measurement Workshop, Peebles, Scotland*, 1994.

APPENDIX A - DIAGRAMS OF RESULTS

Fig. A.1 - Results from test 1.

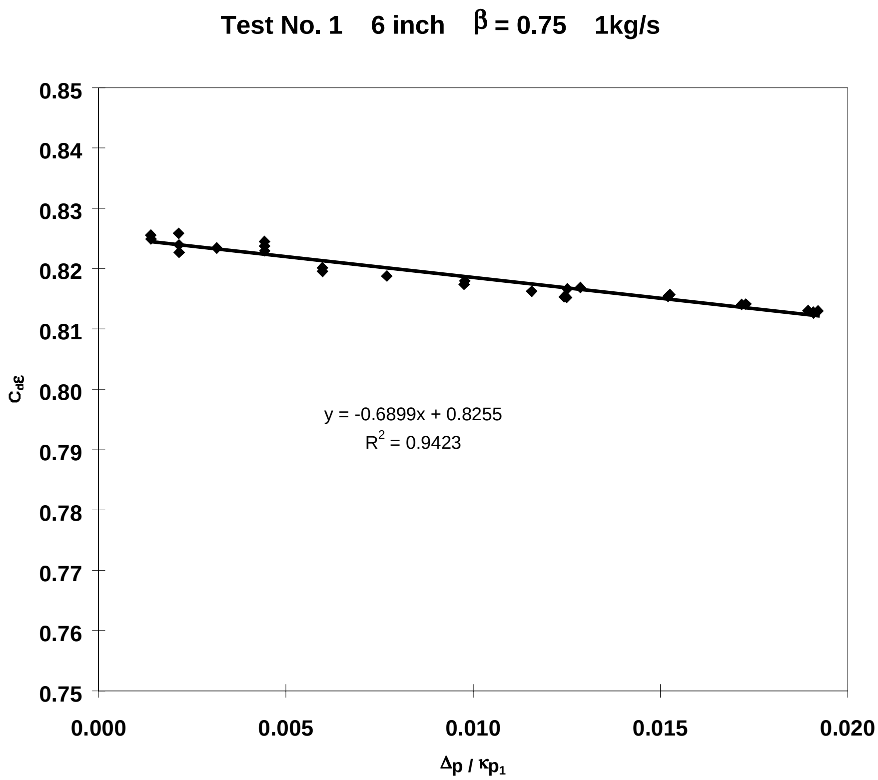

Fig. A.2 - Results from test 2.

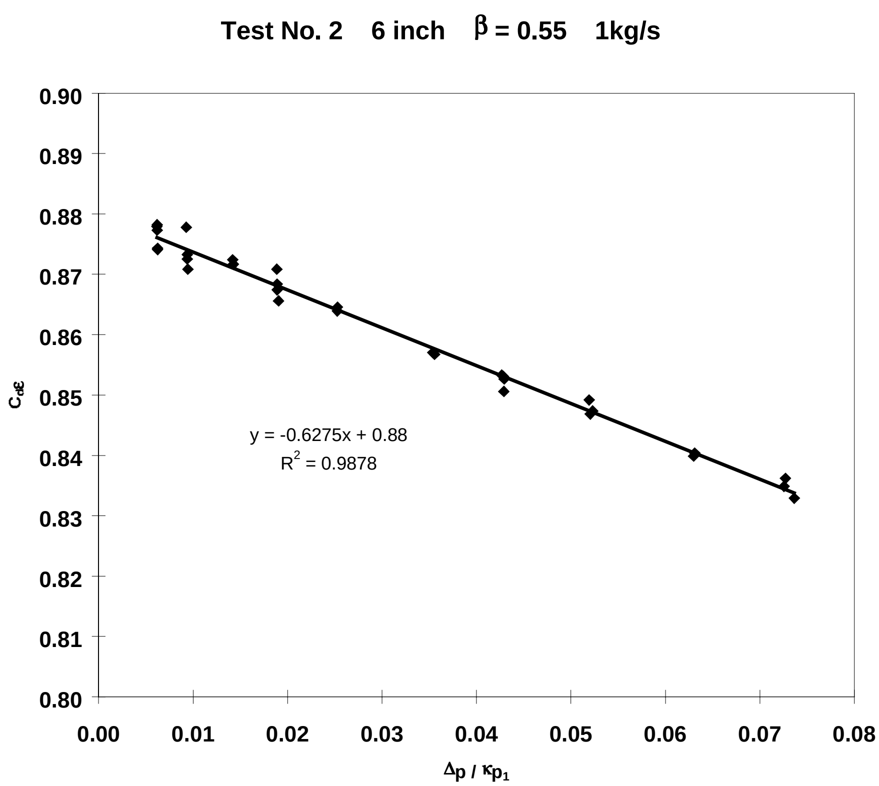

Fig. A.3 - Results from test 3.

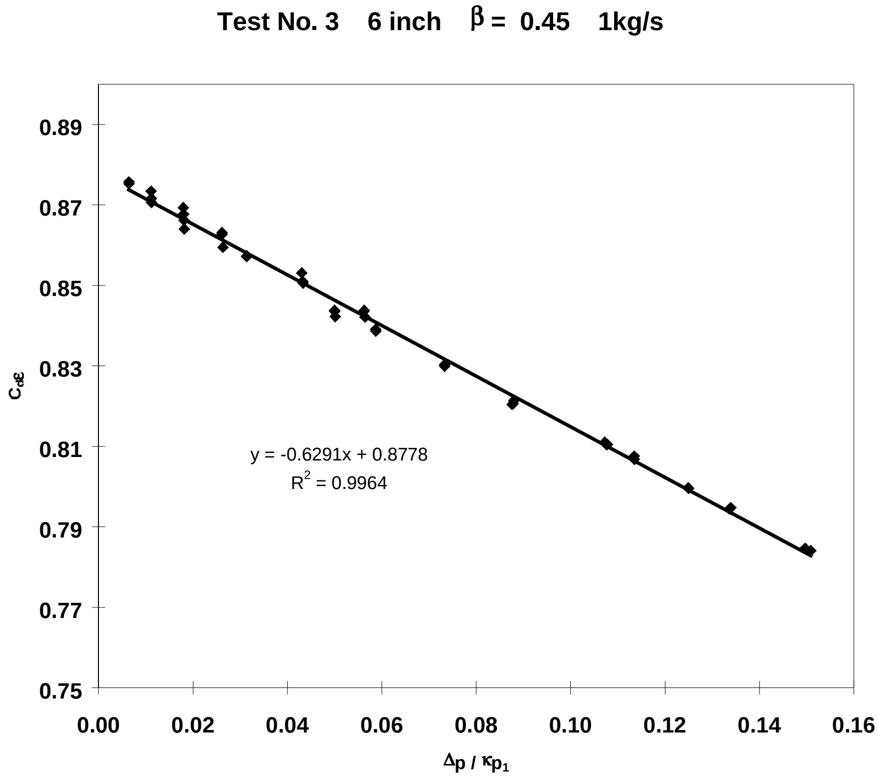

Fig. A.4 - Results from test 4.

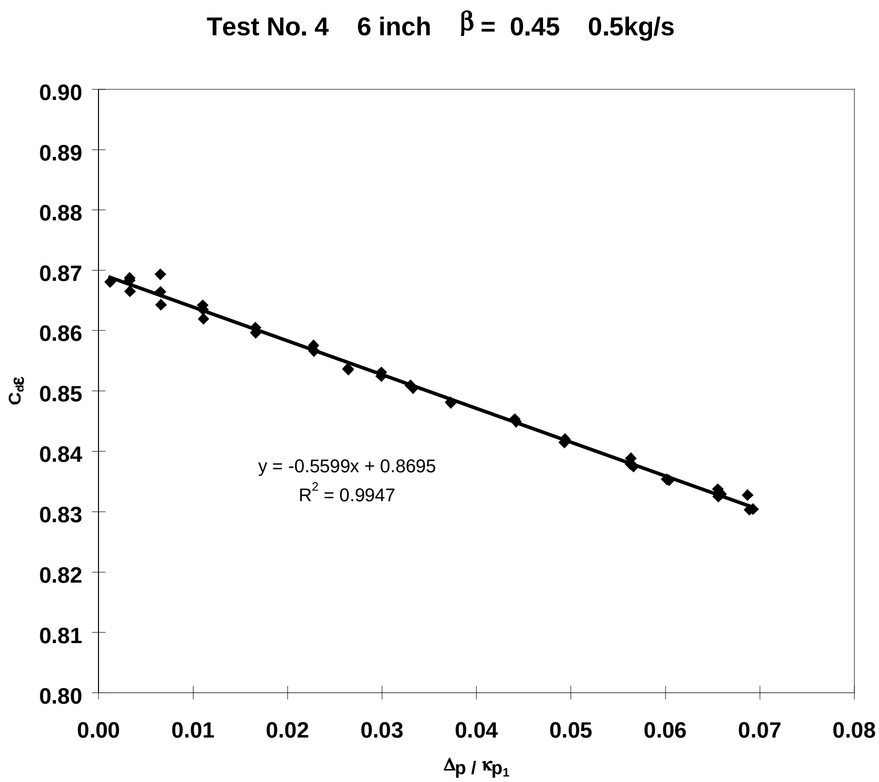

Fig. A.5 - Results from test 5.

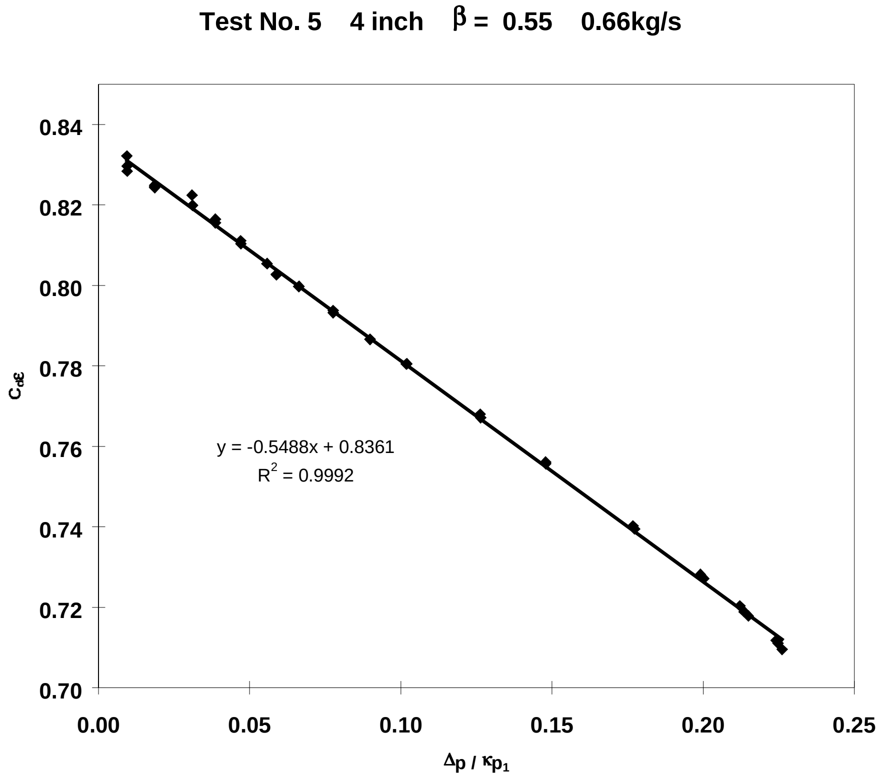

Fig. A.6 - Results from test 6.

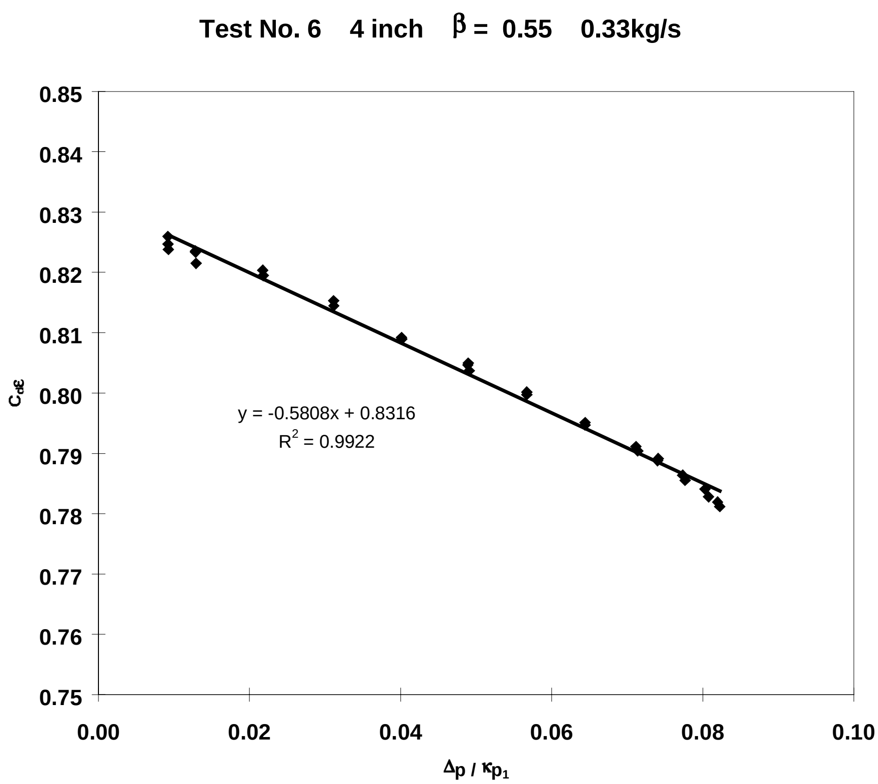

Fig. A.7 - Results from test 7.

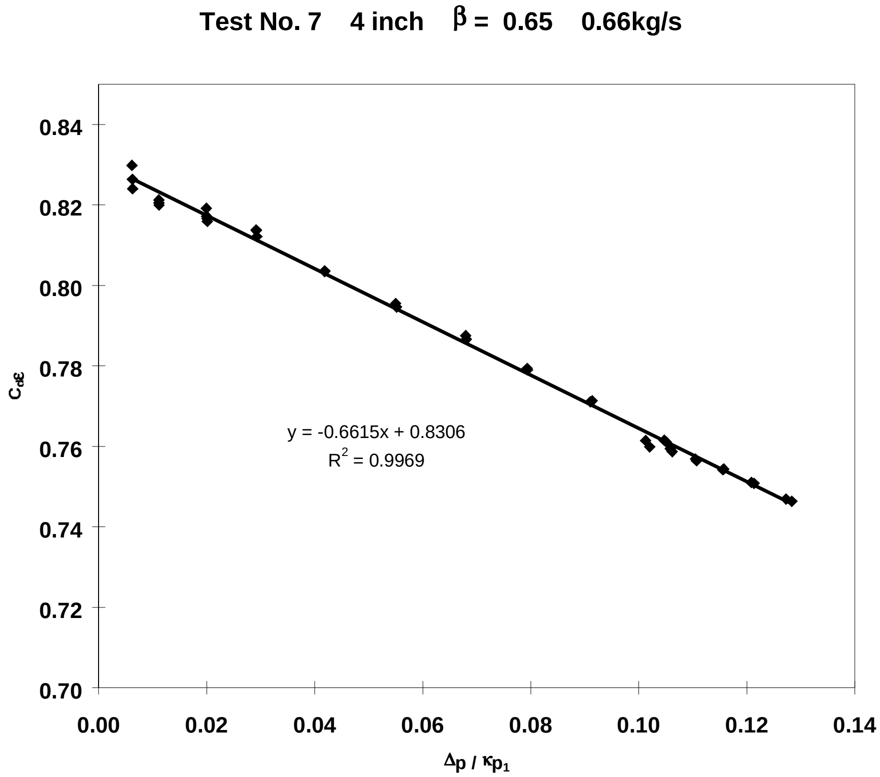

Fig. A.8 - Results from test 8.

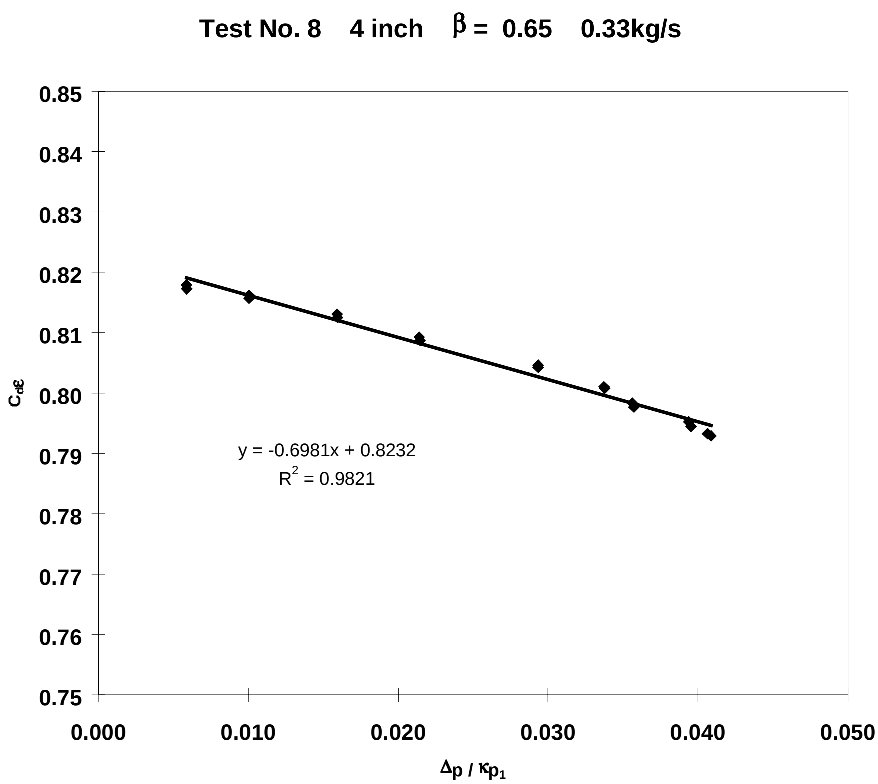

Fig. A.9 - Results from test 9.

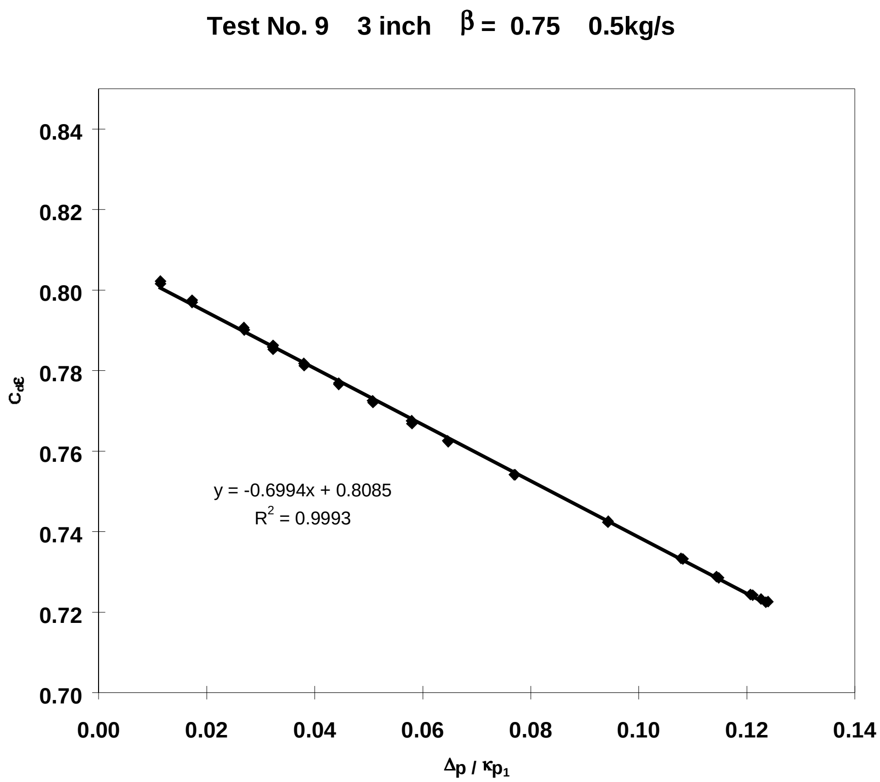

Fig. A.10 - Derived expansibility factor values from test 1 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

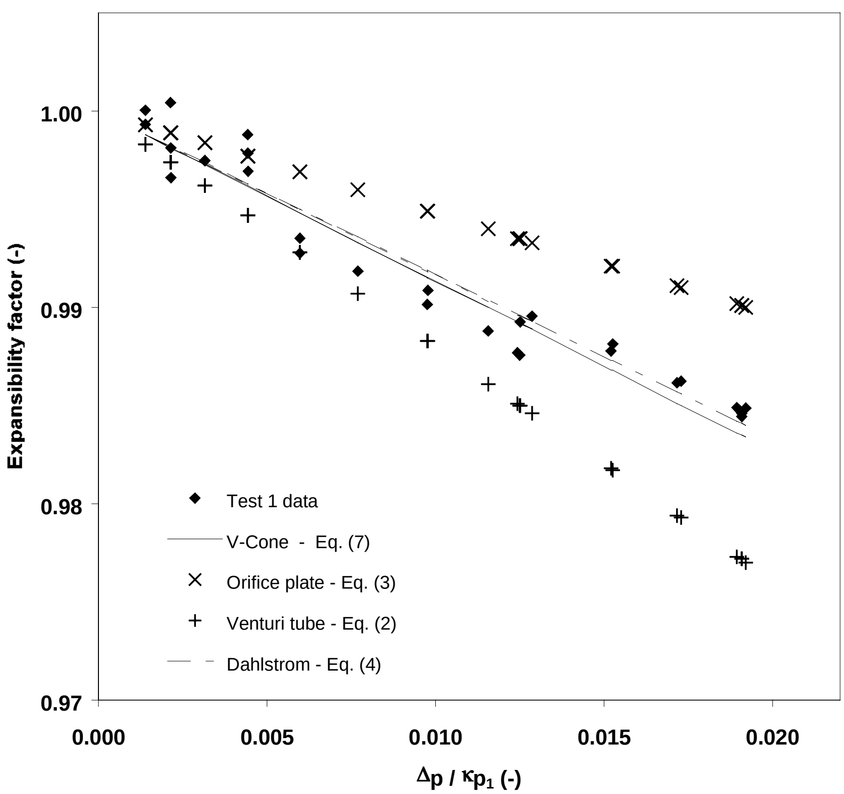

Fig. A.11 - Derived expansibility factor values from test 2 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

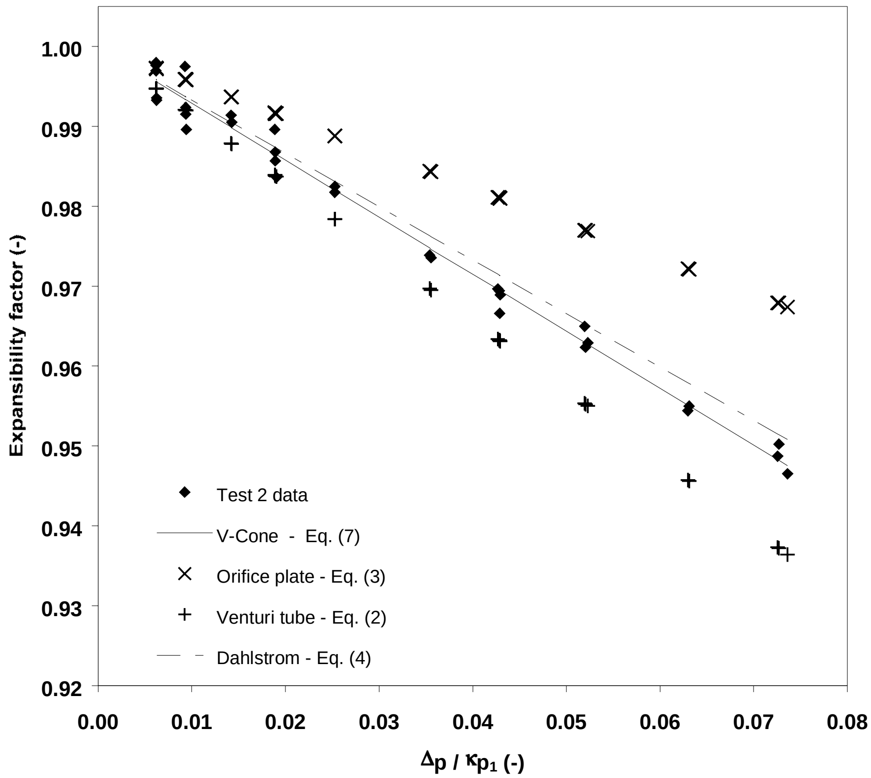

Fig. A.12 - Derived expansibility factor values from test 3 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

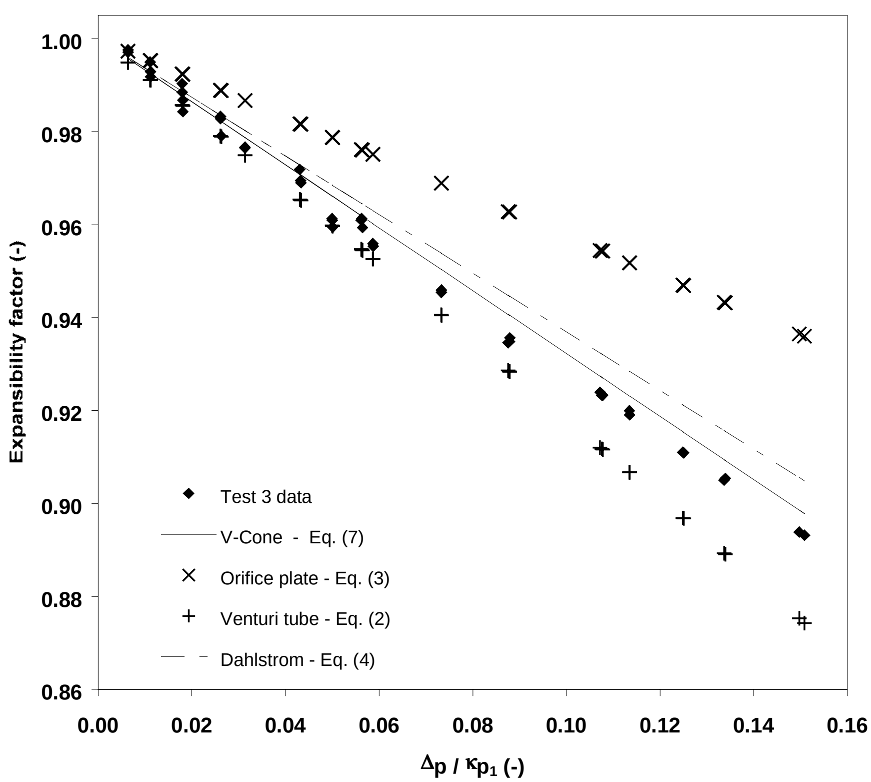

Fig. A.13 - Derived expansibility factor values from test 4 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

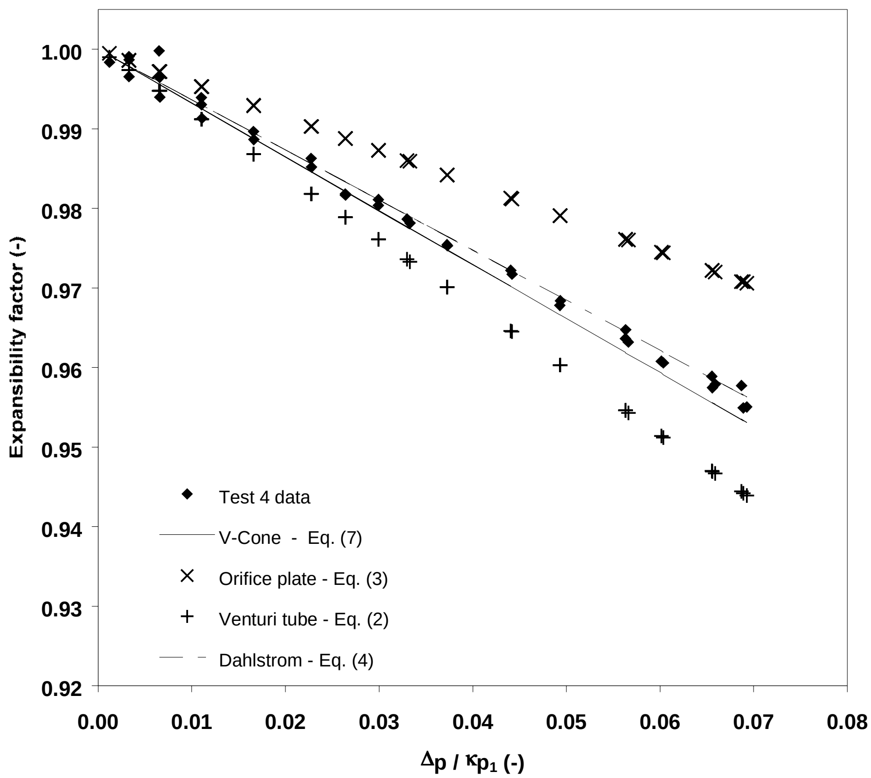

Fig. A.14 - Derived expansibility factor values from test 5 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

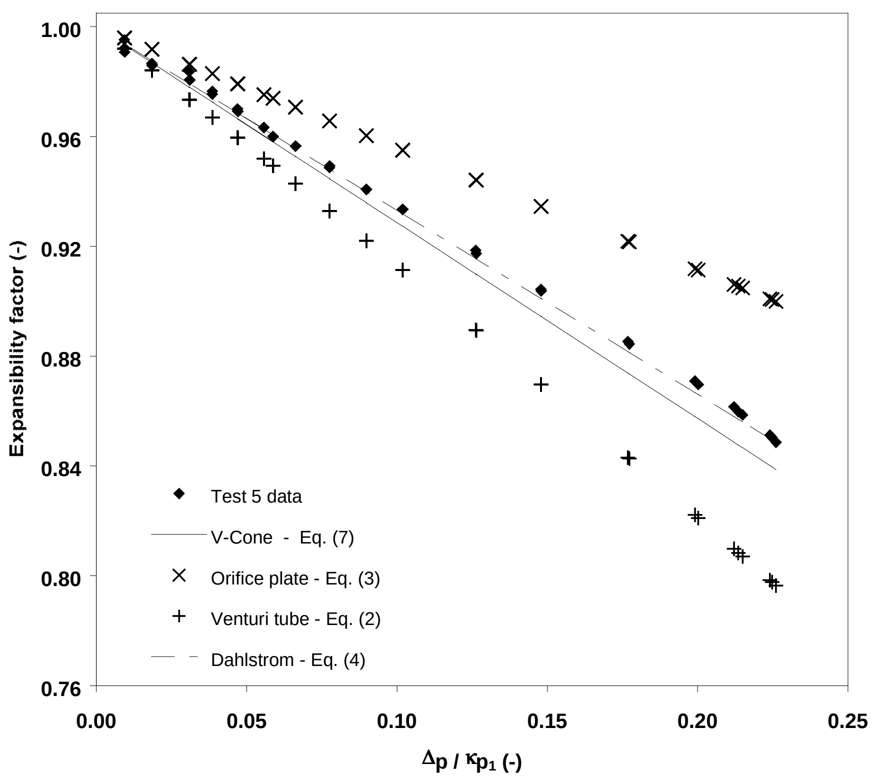

Fig. A.15 - Derived expansibility factor values from test 6 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

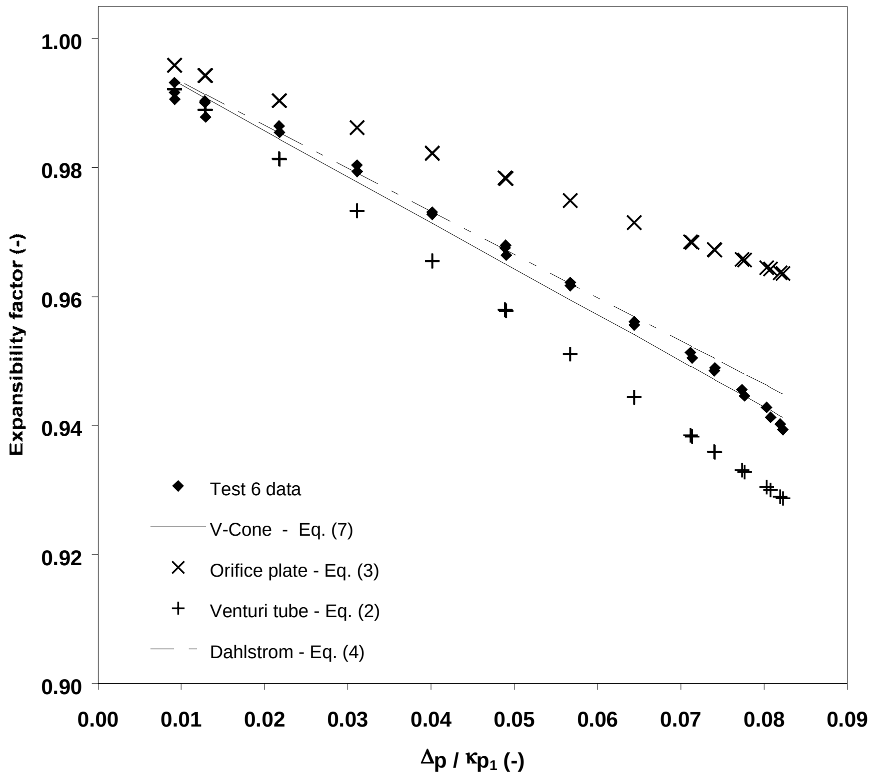

Fig. A.16 - Derived expansibility factor values from test 7 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

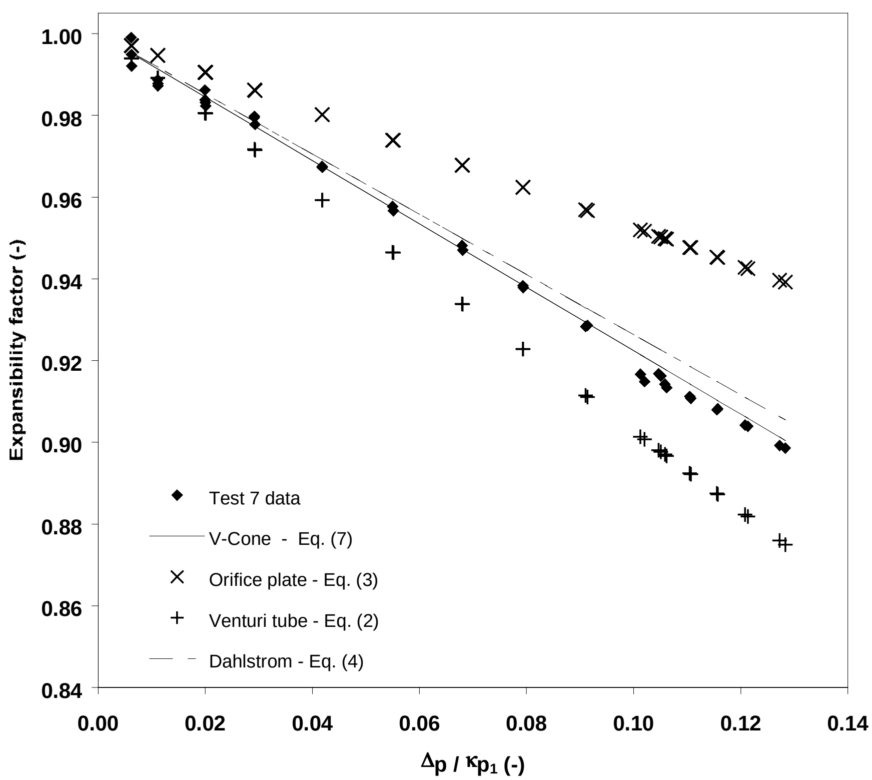

Fig. A.17 - Derived expansibility factor values from test 8 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.

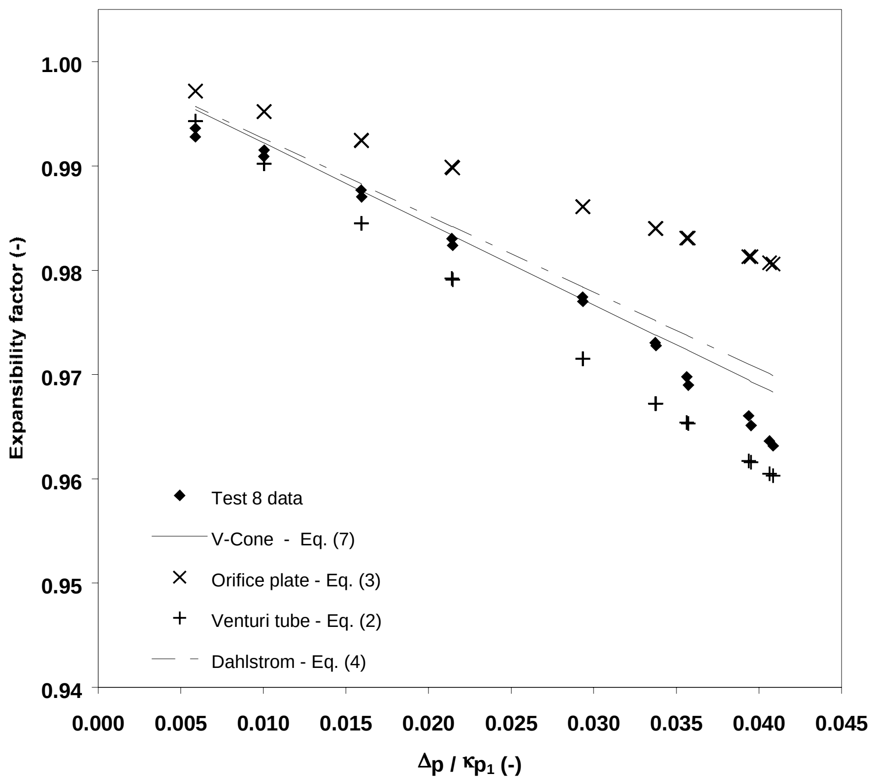

Fig. A.18 - Derived expansibility factor values from test 9 compared with Eq. (7) along with Dahlstrom's equation, and the orifice plate and Venturi tube equations.